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# SELF-ORGANIZATION AND MULTIFRACTALITY IN INFLATION AND PRICE SYSTEMS

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RESUMEN: El análisis de los sistemas de precios como sistemas complejos es de suma importancia para entender la asignación de recursos en la economía a partir de las interacciones entre agentes. En este trabajo se analiza el sistema de precios de la economía uruguaya utilizando como insumos el índice de precios al consumidor desagregado a nivel de producto y el índice general. Del análisis de la distribución de las variaciones de precios en cada periodo y de las variaciones generales de precios y realizando un análisis multifractal, se obtienen resultados robustos y consistentes en el sentido de entender las variaciones de precios como provenientes de sistemas complejos. Los principales resultados indican que no se rechaza la hipótesis de una ley de potencias como la distribución de la serie analizada, así como el comportamiento persistente de las variaciones de precios se ve modificado por grandes fluctuaciones en el sistema.

*Palabras Clave*: Inflación; sistemas complejos; distribución power-law; auto-organización; análisis multifractal.

ABSTRACT: The analysis of price systems as complex systems is of utmost importance to understand the allocation of resources in the economy based on the interactions between agents. In this paper, the price system of the Uruguayan economy is analyzed using the consumer price index disaggregated at the product level and the general index as inputs. From the analysis of the distribution of the price variations in each period and the general price variations and performing a multifractal analysis, we obtain robust and consistent results in the sense of understanding the price variations as coming from complex systems. Main results indicate that the hypothesis of a power-law as the distribution of the analyzed series is not rejected, as well as the persistent behavior of price variations is modified by large fluctuations in the system.

*Keywords*: Inflation; Complex Systems; power-law distribution; self-organization; multifractal analysis.

## 1. Introduction

The economy is an open system: it is formed by a large number of economic agents and the environment. Individuals react to external stimuli (from the environment and other individuals) and internal stimuli in a self-organized way (Mantegna & Stanley, 1999). Self-organized criticality (SOC) (Bak et al., 1988; Bak & Chen, 1991) supposes that open systems with elements that interact with each other are organized in the environment of a "stationary" critical state, with no other scales other than those imposed by the size of the system. These critical states are characterized by temporal and spatial power laws. The density function of a power-law can be defined as follows:

$$f(x) = Cx^{-\alpha} \text{ ; con } \{\alpha, C\} > 0$$

and its $k$ moments as:

$$\langle x^k \rangle = \int_{x_{min}}^{\infty} x^k f(x)dx \text{ ; with } \alpha > k + 1$$

Once one can establish that if data is distributed as a power law, then using the previous formula the first $(\alpha - 1)$ moments can be computed. Then, there are examples in Economics where the variance is asymptotically infinite, such as Zipf's law $(\alpha \simeq 2)$, or income distribution $(2 < \alpha < 3)$[1].

One type of economic phenomenon that could be characterized in this manner is price variations in different markets. These price variations reflect on inflation, which is measured through price indexes (CPI indexes), which can inherit many of the characteristics of the underlying processes. One of the ways in which these characteristics can manifest is through a long memory process with unbounded shocks. On the one hand, there is evidence for different economies that states that the distribution of price variation at the product level is skewed (Bryan & Cecchetti, 1999; Kearns et al., 1998; Scharnagl & Stapf, 2014). Other authors, such as Tohmé et al. (2005) conclude that for some Latin American countries, the distribution of the price variation follows a power law. Others, such as Boubaker et al. (2017) or Fernandes et al. (2020), analyze the persistence parameters of inflationary processes based on the Hurst exponent. As mentioned by Fernandes et al. (2020), long memory processes are associated with a history of indexing in countries with a history of high inflation.

This article analyzes if the evolution of prices in the economy can be analyzed only from the mean and the variance of the price index, based on the example with data from Uruguay. Therefore, three complementary objectives are presented. In the first part, the distribution of the price variations of the products that belong to the CPI index is analyzed, in order to know for each unit of time how this random process can be represented. Next, we will study how the variations in the general price index are distributed over time. It can be analyzed in a similar way to other financial assets, although on the other hand, consumer price indexes are an important input for the functioning of the economy, especially in the case of economies with a high degree of indexation. Finally, we will analyze if these series presents persistent shocks or if instead, shocks tend to reverse over time. For this, we will control persistence according to the size of the shock and the time window analyzed, using multifractal analysis. The empirical results show that the hypothesis of a power-law as the distribution of the analyzed series is not rejected, while this analysis allows rejecting other hypotheses regarding the underlying distribution in the price series, with implications in the way that researchers make assumptions about the volatility of price series. Another important result is that the persistence of price variations is not constant, but depends on the size of the fluctuations in the system. The paper continues as follows. Section 2 details the

[1] See Gabaix (1999), Drăgulescu & Yakovenko (2001) and Clementi & Gallegati (2005).

methodology and data used. In section 3 the most important results of this study are analyzed and in section 4 the conclusions of this work are discussed.

## 2. Methodology

For this article, we analyze the empirical distribution of price variations over time and of a price group at each moment. In particular, it is tested whether the empirical distributions found can be represented from a power-law, as in Equation 1 using maximum likelihood, as in Tohmé et al. (2005) and Alvarez & London (2020). Next, the multi-fractal dimension of the aggregate CPI series is analyzed.

$$\begin{aligned} H_0 &: X_t \sim PL(\alpha, x_{min}) \\ H_1 &: X_t \nsim PL(\alpha, x_{min}) \end{aligned} \tag{1}$$

The Hurst exponent $h$ (Hurst, 1951) allows to estimate the degree of temporal dependence of the series. The higher the value of this exponent, the behavior of the series is more regular and less erratic than with lower values. Values greater than 0.5 for this exponent imply a high temporal dependence, persistent processes, or "long-term memory", while values below 0.5 are anti-persistent processes or "pink noise". The special case of $h = 0.5$ corresponds to the absence of autocorrelation, also called "white noise".

The generalized Hurst exponent - which in the literature appears as $h(q)$, $h(q, s)$ - is an extension of the Hurst exponent, whose parameter $q$ refers to the $q$ moments of the distribution of variations. Unlike $h$, in $h(q)$ the different moments have different scales. With $q$ rescaled, we will see that $q$ depends on the size of the shock, with $q < 0$ small fluctuations and $q > 0$ relatively large fluctuations. The parameter $s$ accounts for the timescale. The degree of persistence of the series is not invariant in time but depends on the time scale used.

The series of the Consumer Price Index (CPI) generated by the National Institute of Statistics of Uruguay is used, with aggregated and disaggregated data. The series of aggregated data corresponds to the global CPI, for which there is information from July, 1937 to December, 2020. To know the behavior of the price distribution within each month, we use this indicator disaggregated at the product level. For this article, the series are used without correction for seasonality. The problem of the seasonality of the series is especially relevant since the fluctuations of many of these prices have intra-annual patterns. For many applications, it may be valid to obtain a seasonally adjusted measure of price movements, for example for forecasting purposes; however, in this exercise, it is important to use the series without seasonal adjustment, to capture the volatility within the CPI. This high price volatility generates major problems at the aggregate level; this volatility depends in turn on seasonal factors, shocks to products, as well as systemic factors, which affect a large number of components of the index simultaneously. In turn, due to the interrelation between the components of the index, there is a contagion effect.

Based on the year 2010, we have a basket of 374 products from December, 2010 to December, 2020. The data is applied as of December 2010 because there was a methodological change in the index as of that date, and it is not possible to combine the values before that date since the categories do not coincide in many cases. In any case, these calculations could be made for the period 1997–2010, but for the comparison of both periods, the changes in the composition of the index must be taken into account. For the period starting in 2020, in Alvarez et al. (2022) it is observed that the dynamics of the price network of the economy is modified; it is found that the behavior of this network in that period obeys the characteristics of a systemic shock. This shock, however, is consistent with the type of shocks in relative prices that have occurred in the period analyzed and does not modify the global results. First differences are applied to the original series, both to the aggregated and disaggregated data.

This methodology is applied to analyze, in the current context, the scope of the inflationary shocks on the prices that make up the index. This methodology, applied in Tohmé et al. (2005) and Fernandes et al. (2020), is important because it allows us to observe the probability associated with the shocks in each price and also allows us to observe a generative model of a power law in the general index from the aggregation of multiple distributions that follow a power law. The values associated with this distribution allow us to know the systemic nature of the deviations; in this case, the extreme values generate changes in the calculation of the index and the relative prices. On the other hand, the nature of the volatility found allows us to establish that this volatility increases with the size of the sample of products in the index. Under other econometric techniques, we seek to capture the inertial or cyclical component of the time series, while in this case, we focus on the problems derived from the aggregation of highly volatile time series.

## 3. Results

### 3.1. *Distribution of product price variations at one month*

Firstly, this paper analyzes the distribution of monthly price changes for each product. For a database of 374 articles, we analyzed the distribution within each month and the data pool within each year. As compared with the empirical distributions resulting from a power law, the cumulative distribution is:

$$F(x) = P(X > x) = a.x^{(1-\alpha)} \tag{2}$$

Figure 1 shows that within each year and each month, in log-log scale it can be observed a decreasing trend, which is consistent with a power law distribution.

The coefficient $\alpha$ and the $p - value$ of the statistic are estimated by maximum likelihood for each month and each year. As the main result, we can say that the hypothesis of a power-law is not rejected in the data generating process.

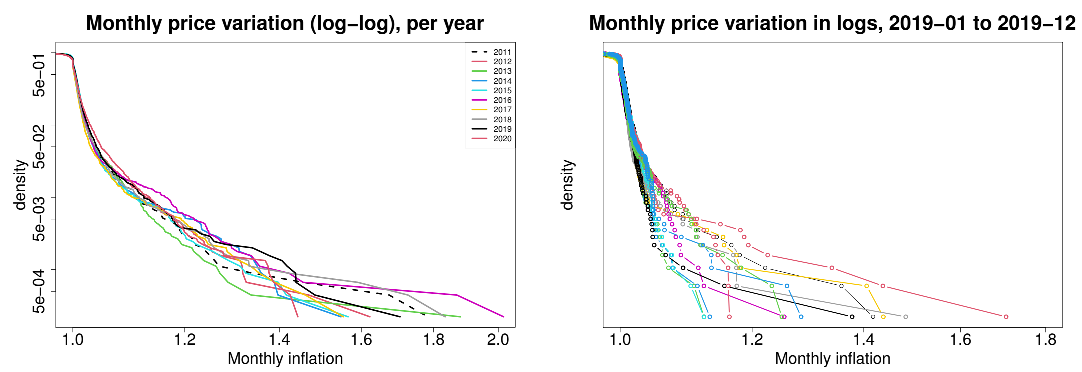


Figura 1. Left: Distribution of price variation at the level of CPI products, data from 2011 to 2020. Right: Distribution of price variation at the level of CPI products, January to December 2019. Source: own construction based on data from National Institute of Statistics of Uruguay.

It is found that for a significance level of 5 %, the null hypothesis of a power-law in 98 % of months analyzed (Figure 1, right) is not rejected, while this hypothesis is not rejected for the years

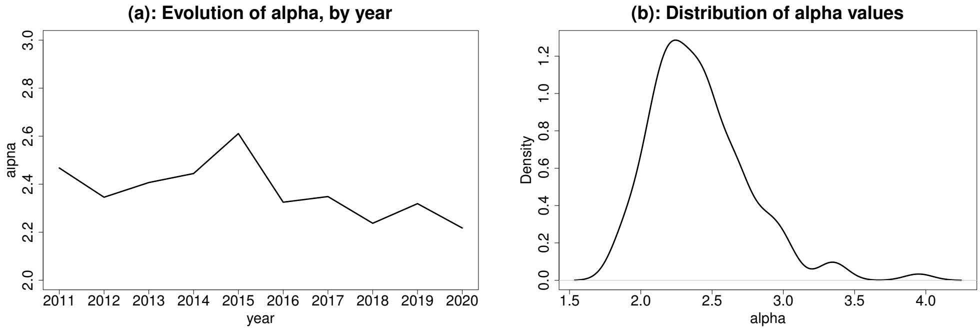


Figura 2. (a): Parameter $\alpha$, from 2011 to 2020; (b): Distribution of $\alpha$ from January 2011 to December 2020. Source: own construction based on data from National Institute of Statistics of Uruguay.

2011 to 2020 (Figure 1, left)[2].

An important aspect of these results, in addition to the non-rejection of a power-law distribution, is the value found for the parameter $\alpha$. As shown in Figure 2, the value of $\alpha$ is decreasing in time and $2 < \alpha < 3$. The first moment of the distribution found is finite, while the second moment of the distribution is (asymptotically) infinite. This result indicates that the moments of the distribution are not stable, with the consequence that a shock can generate unbounded deviations. One interpretation of this result is that this technique perceives high volatility in the variation of product prices. Changes in relative prices associated with this volatility are not captured by the variations in the headline index, although they have severe impacts on the different markets. The asymptotically infinite behavior of the second moment of the distribution of price changes has another important consequence: the volatility of the index increases with the number of products in the index. This result, although it seems counterintuitive, is derived from the previous result ($\alpha < 3$). This is explained because the greater the number of products, the greater the chances of capturing large deviations in some of the products belonging to the index increase. On the other hand, it can be seen that the value of $\alpha$ has decreased in the last 10 years. This phenomenon accounts for the greater volatility of relative prices in the economy and, even more importantly, it increases the exposure to significant shocks in the price index.

### 3.2. *Distribution of the headline CPI series*

n this section, we propose to analyze the distribution of the series of the first differences in the aggregate price index. This index, due to the highly indexed nature of the Uruguayan economy, is used to update salaries, liabilities, and financial contracts. It is therefore important to know how shocks can affect the price level of the economy or some of these prices. The price index in Uruguay has different phases (see Figure 3(a)), which correspond to the different stabilization periods. In turn, as can be seen in Figure 3(b), in terms of monthly variations, this series is influenced by an important seasonal component.

There is evidence for Latin America (Caraballo & Dabus, 2005) that the distribution of price variation in an economy can be formulated from a power law, while there is evidence for Brazil (Fernandes et al., 2020) that the price index can be represented from its fractal dimension.

When analyzing the distribution of this series in first differences, we verify that we do not reject the null hypothesis of a power law, both for the complete series, for different time intervals as well

[2]As exceptions in this decade of data, only August-2013 and May-2019 are found. Other distributions, such as normal, log-normal, or exponential are rejected for all periods analyzed.

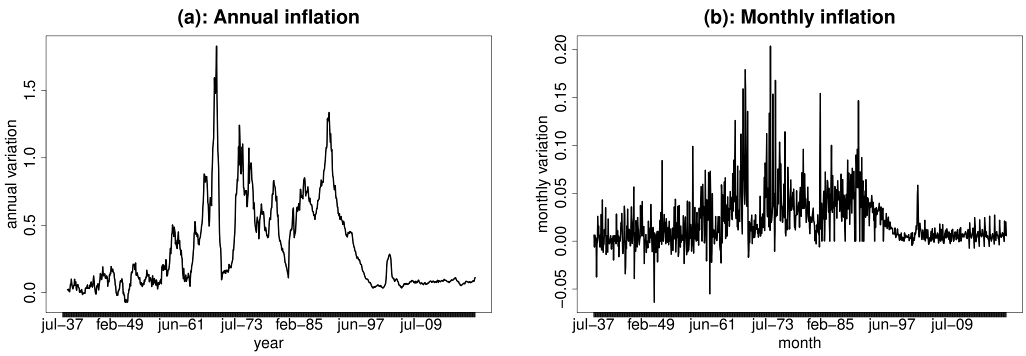


Figura 3. CPI data in first differences, July, 1937 to December, 2020. (a): annual inflation; (b): monthly inflation. Source: own construction based on data from National Institute of Statistics of Uruguay.

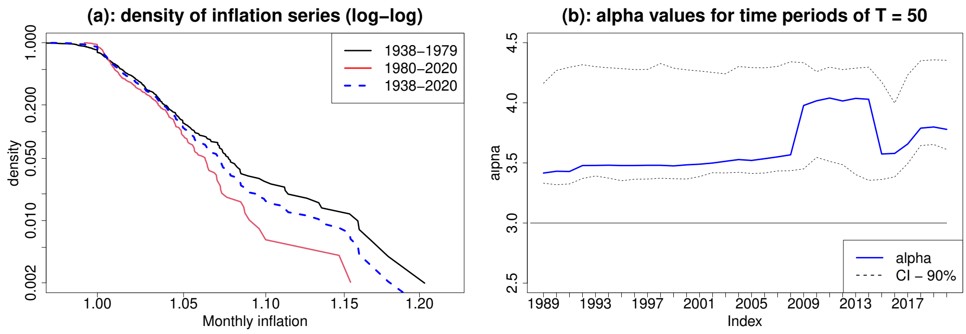


Figura 4. (a): graph of the density of CPI series in first differences, on a logarithmic scale; (b): value of the alpha parameter using time windows of 50 years. Source: own construction based on data from National Institute of Statistics of Uruguay.

as for different time windows.

In Figure 4(a) it can be seen that on a logarithmic scale, both the complete series and in sub-periods of 40 years have a behavior similar to that expected before power-law processes. It is also observed that in recent years a lower probability is assigned to higher monthly variations, which corresponds to lower volatility and a higher value of the parameter $\alpha$. It is observed in Figure 4(b) that in time windows of 50 years, this parameter is increasing, with values greater than 3. What is observed in Figure 4 is consistent with the moderation of inflationary trends in recent decades.

This result of the $\alpha$ parameter indicates that the general price index is less volatile than the products that compose it. This result has at least two explanations: on the one hand, the aggregate index compensates for price variations at the micro-level, which is why volatility is lower. However, as we saw previously, the intrinsic volatility of the monthly data means that there is a greater degree of uncertainty in the aggregate series. On the other hand, unlike product prices, headline inflation is a monetary policy objective. The Central Bank seeks to intervene in the different markets to reduce both the variation and the volatility of the index, but this does not seem to affect the volatility of monthly price variations.

The volatility of the index is lower than that calculated for the financial series (see Mandelbrot & Taylor (1967)), although a comparison between the financial series and weighted averages of the price series should be taken with caution.

The lower volatility of the CPI can be understood by its condition as an instrument and the

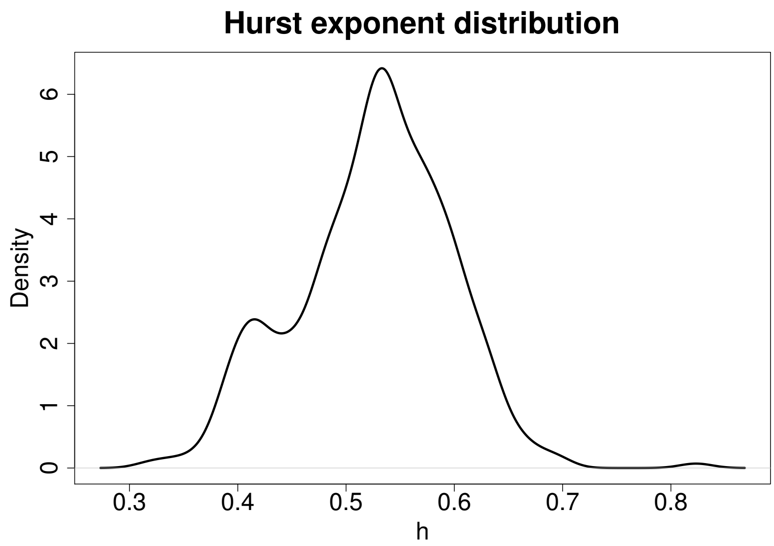


Figura 5. Distribution of the Hurst exponent for the products of the CPI. Source: own construction based on data from National Institute of Statistics of Uruguay.

objective of economic policies. On the one hand, low and stable inflation is the objective of Central Bank policies; this index is also the focal point of many policies, and the expectations of many economic agents are linked to the variation of this index. As a result, different anti-inflationary policies seek to influence the behavior of the index, be it monetary, fiscal, or restricting prices and/or quantities.

### 3.3. *Hurst exponent for CPI products' price series*

In this section, we propose to analyze the price series through fractal and multifractal analysis. Analyzing the one-dimensional Hurst exponent for the series of the CPI products, it is observed that not all the price series show the same behavior.

As can be seen in Figure 5, the distribution is bimodal, where it can also be seen that 33% of the products have a value of $h < 0.5$, while the remaining 67% have a $h > 0.5$. One factor that can cause this behavior of the price series lies in the possible existence of price groups with different price formation process dynamics that are not captured in the headline index. Therefore, it is possible to extract more information from the analysis of the CPI from sub-indexes, with greater intra-group homogeneity and greater heterogeneity between groups. For example, in the economic literature, the categorization of tradable and non-tradable prices is used, since the former depends to a greater extent on what happens in the rest of the world, while the latter is more related to events in the domestic economy. In this regard, there is important evidence at the international level (see De Gregorio et al. (1994)), arising from the models of Balassa (1964) and Samuelson (1964). There are some empirical approaches to this type of categorization for Uruguay, see Cancelo et al. (1994); Alvarez et al. (2021).

### 3.4. *Generalized Hurst exponent for the CPI series in first differences*

Figure 6 represents the values of the generalized Hurst exponent over the last 50 years of data, taken from the headline index. It is observed that this function is decreasing in $q$: it implies that in the event of minor deviations the series is persistent, while in large deviations the series tends

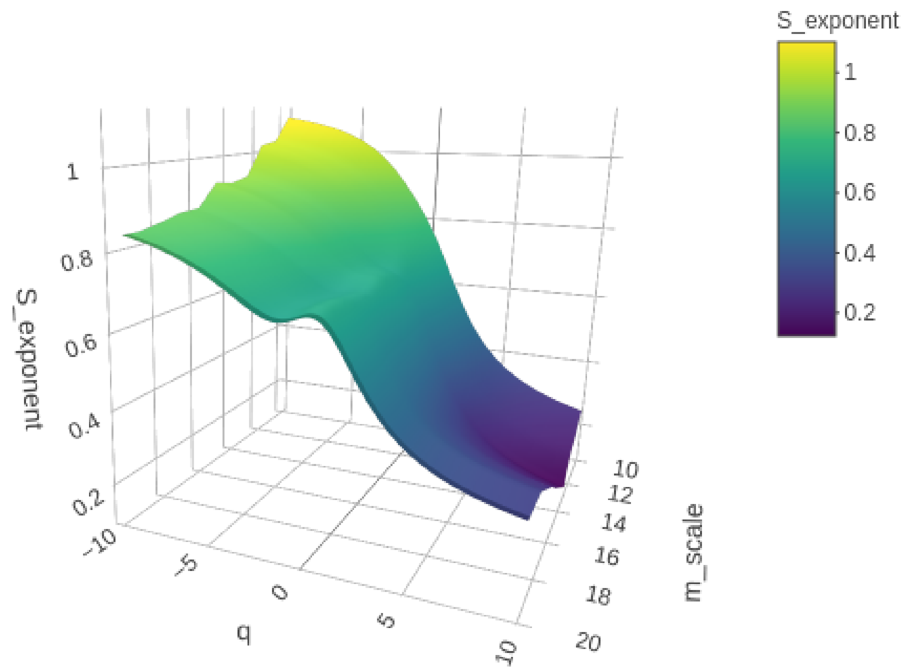


Figura 6. h(q,s) plot for monthly inflation, from January, 1970 to December, 2020. Source: own construction based on data from National Institute of Statistics of Uruguay.

to revert to the mean. In the same way, we can see that it does not depend on the $s$ parameter, which refers to the timescale. As can be seen in Figure 6, the behavior of this series is similar under different time scales.

Based on the result of Figure 6, we can rewrite the Hurst exponent as $h = h(q)$, that is, the value of the exponent depends on the size of the shock. However, the values of this exponent may be contaminated by seasonality and autocorrelations of the original series. To eliminate this possible source of error, we follow two procedures. On the one hand, following Fernandes et al. (2020), the same calculation is performed with the shuffled series, in order to eliminate the autocorrelations of the series. On the other hand, the X-13 ARIMA-SEATS Seasonal Adjustment Program[3] is used as a filter for seasonal adjustment. The $h(q)$ results for these series are shown in Figure 7. In Figure 7 it can be seen that even for the shuffled series or the seasonally adjusted series, $h(q)$ behaves similarly to the original series, although with a lesser difference in $h(q)$ before differences in the size of the deviations $q$. In the case of small or medium-large deviations ($q < 5$ approx), in all the cases analyzed the system behaves persistently, while for large deviations the system tends to revert to the mean.

## 4. Conclusion

In this article, the distribution of the Uruguayan CPI price series has been analyzed using different techniques, both in aggregate and disaggregated form. The first conclusion offered by this study is that for the different scales and periods analyzed, we cannot reject the hypothesis of a power-law

[3]https://www.census.gov/data/software/x13as.html

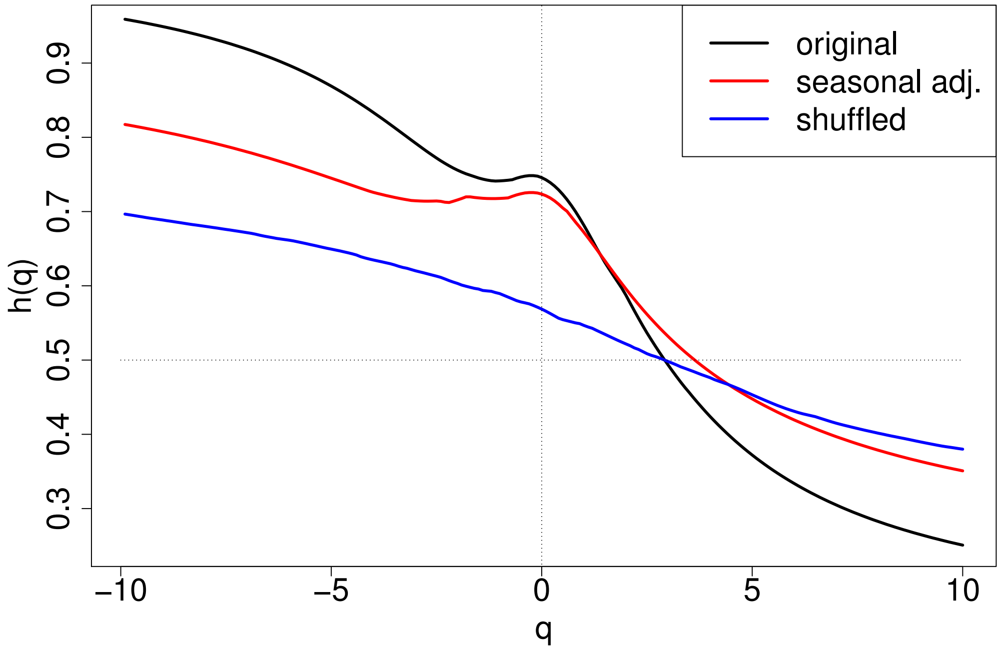


Figura 7. H(q) of the original, seasonal adjusted and shuffled series. Data from January, 1970 to December, 2020. Source: own construction based on data from National Institute of Statistics of Uruguay.

as the distribution of the analyzed series, while this analysis allows rejecting other hypotheses regarding the underlying distribution of the price series.

The power-law distribution found has important consequences from the economic and prediction points of view, in particular in the analysis of volatility. The volatility of the intra-month product price variation has consequences for the degree of uncertainty of the point estimate of the aggregate indicator, as well as the possibility of large fluctuations in the relative prices of the economy. The inflationary process in Uruguay has periods of relatively high inflation and high volatility; this analysis shows that there is a non-zero probability of large fluctuations in the price variations of the economy. These fluctuations are not white noise, but their behavior (persistent or reverting to the mean) depends on the entity of the shock. This result is also generated from the interactions between agents and their interventions in the environment to influence the behavior of the index.

On the other hand, the information provided by the distribution of $h(q)$ is analogous to the distribution of the first differences of the CPI, which gives robustness to the results. At the same time, both analyses provide complementary information: on the one hand, we can estimate the probability of an event occurring; on the other hand, we can analyze if this event will be persistent over time or if the system will return to the parameters before the shock. These results are robust as they offer additional evidence to Fernandes et al. (2020) in the case of developing countries with significant indexation.

We consider that this study can be extended in other directions. For example, this analysis can be carried out for other economic series in Uruguay related to inflationary processes (value of other currencies, terms of trade, real variables).

Finally, it may be relevant to establish relationships between the behavior of agents' inflation expectations and the price formation process in the economy, based on the assumption that both are in a SOC.

### *References*


Alvarez, E., Brida, J. G., Martínez, M., & Mones, P. (2022). Análisis de redes complejas: un estudio de la inflación en Uruguay. *Revista Finanzas y Política Económica*, 14(1), 131–166.

Alvarez, E., Brida, J. G., & Mones, P. (2021). Dinámica de la estructura de precios en Uruguay. *Rect@: Revista Electrónica de Comunicaciones y Trabajos de ASEPUMA*, 22(1), 1–19.

Alvarez, E. & London, S. (2020). Emerging patterns in inflation expectations with multiple agents. *Journal of Dynamics & Games*, 7(3), 175.

Bak, P. & Chen, K. (1991). Self-organized criticality. *Scientific American*, 264(1), 46–53.

Bak, P., Tang, C., & Wiesenfeld, K. (1988). Self-organized criticality. *Physical review A*, 38(1), 364.

Balassa, B. (1964). The purchasing-power parity doctrine: a reappraisal. *Journal of Political Economy*, 72(6), 584–596.

Boubaker, H., Canarella, G., Gupta, R., & Miller, S. M. (2017). Time-varying persistence of inflation: evidence from a wavelet-based approach. *Studies in Nonlinear Dynamics & Econometrics*, 21(4).

Bryan, M. F. & Cecchetti, S. G. (1999). Inflation and the distribution of price changes. *Review of Economics and Statistics*, 81(2), 188–196.

Cancelo, J. R., Fernández, A., Grosskoff, R., Selves, R., & Villamonte, G. (1994). Precios de transables y no transables: Un enfoque arima-ia. *IX Jornadas de Economía del Banco Central del Uruguay, Montevideo, Uruguay.*

Caraballo, M. & Dabus, C. (2005). Nominal rigidities, relative prices and skewness. *Centro de Estudios Andaluces, Working Paper series.*

Clementi, F. & Gallegati, M. (2005). Pareto's law of income distribution: Evidence for germany, the united kingdom, and the united states. In *Econophysics of wealth distributions* (pp. 3–14). Springer.

De Gregorio, J., Giovannini, A., & Wolf, H. C. (1994). International evidence on tradables and nontradables inflation. *European Economic Review*, 38(6), 1225–1244.

Drăgulescu, A. & Yakovenko, V. M. (2001). Exponential and power-law probability distributions of wealth and income in the United Kingdom and the United States. *Physica A: Statistical Mechanics and its Applications*, 299(1-2), 213–221.

Fernandes, L. H., Araújo, F. H., Silva, I. E., Leite, U. P., de Lima, N. F., Stosic, T., & Ferreira, T. A. (2020). Multifractal behavior in the dynamics of brazilian inflation indices. *Physica A: Statistical Mechanics and its Applications*, 550, 124158.

Gabaix, X. (1999). Zipf's law for cities: An explanation. *The Quarterly Journal of Economics*, 114(3), 739–767.

Hurst, H. E. (1951). Long-term storage capacity of reservoirs. *Transactions of the American society of civil engineers*, 116(1), 770–799.

Kearns, J. et al. (1998). Measuring core inflation— rdp 9810: The distribution and measurement of inflation. *Reserve Bank of Australia Research Discussion Papers*, (September).

Mandelbrot, B. & Taylor, H. M. (1967). On the distribution of stock price differences. *Operations Research*, 15(6), 1057–1062.

Mantegna, R. N. & Stanley, H. E. (1999). *Introduction to econophysics: correlations and complexity in finance.* Cambridge University Press.

Samuelson, P. A. (1964). Theoretical notes on trade problems. *The Review of Economics and Statistics*, (pp. 145–154).

Scharnagl, M. & Stapf, J. (2014). Inflation, deflation, and uncertainty: What drives euro area option-implied inflation expectations and are they still anchored in the sovereign debt crisis? *Bundesbank Discussion Paper*.

Tohmé, F., Dabús, C., & London, S. (2005). Processes of evolutionary self-organization in high inflation experiences. In *New Tools of Economic Dynamics* (pp. 357–371). Springer.