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SEMIORDERS, INTERVALS ORDERS AND PSEUDO ORDERS PREFERENCE STRUCTURES IN MULTIPLE CRITERIA DECISION AID METHODS

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RESUMEN: Durante las últimas décadas, un importante número métodos multicriterio de ayuda a la decisión (MCDA) ha sido propuesto para ayudar a los decisores en el proceso de selección de alternativas de compromiso. Mientras tanto, el método PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluations) de la familia de los métodos de superación y sus aplicaciones han captado la atención de académicos y profesionales. En este trabajo se presenta una extensión de estos métodos, que consiste en analizar su funcionamiento bajo nuevas estructuras de preferencias (NPS). Las estructuras de preferencia a tener en cuenta son: las órdenes de semi órdenes, órdenes intervalares y las pseudo órdenes. Estas estructuras mejoran notablemente la modelización al aportar más flexibilidad, amplitud y certeza a la formulación de preferencias al tender a abandonar el Axioma de Comparabilidad Transitiva Completa de Preferencias por Axioma de Comparabilidad Parcial de Preferencias. Debe ser destacada la introducción de las relaciones de incomparabilidad en el análisis y la consideración de estructuras de preferencias que aceptan la intransitivad de la Indiferencia. La incorporación de la nueva estructura de preferencias se lleva a cabo en tres fases incorporadas en la metodología PROMETHEE: enriquecimiento de la estructura de preferencias, enriquecimiento de la relación de dominación y explotación de las relaciones de superación de ayuda a la decisión, con el fin de llegar finalmente a la solución de ordenación de las alternativas del problema a través del uso de PROMETHEE I o el II PROMETHEE, en función de si se requiere, respectivamente, una clasificación parcial o completa bajo la nueva estructura de preferencias.

Palabras clave: Orden intervalar, semi orden, pseudo orden, relaciones de superación, criterios generalizados, umbrales, métodos PROMETHEE

ABSTRACT: During the last decades, an important number of Multicriteria Decision Aid Methods (MCDA) has been proposed to help the decision maker to select the best compromise alternative. Meanwhile, the PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluations) family of outranking method and their applications has attracted much attention from academics and practitioners. In this paper, an extension of these methods is presented, consisting of analyze its functioning under New Preference Structures (NPS). The preference structures taken into account are, namely: semiorders, intervals orders and pseudo orders. These structures outstandingly improve the modelization as they give more flexibility, amplitude and certainty at the preferences formulation, since they tend to abandon the Complete Transitive Comparability Axiom of Preferences in order to substitute it by the Partial Comparability Axiom of Preferences. It must be remarked the introduction of Incomparability relations to the analysis and the consideration of preference structures that accept the Indifference intransitivity. The NPS incorporation is carried out in three phases that the PROMETHEE Methodology takes in: preference structure enrichment, dominance relation enrichment and outranking relation exploitation for decision aid, in order to finally arrive at solving the alternatives ranking problem through the PROMETHEE I or the PROMETHEE II utilization, according to whether a partial ranking or a complete one, is respectively required under the NPS.

Keywords: Intervals Order, Semiorder, Pseudo Order, Outranking Relations, Generalized Criteria, Threshold, PROMETHEE Methods.

1. Introduction

The PROMETHEE Methods are a very important class of outranking methods in the Multicriteria analysis for decision aid. In the modelization step they use a simple preference structure, that is, the preorder in which the decision maker has only two possibilities to evaluate a pair of alternatives: strict preference or indifference. In this paper, an extension of these methods is presented, consisting of analyze their functioning under New Preference Structures (NPS). The New Preference Structures to take into account are, namely: semiorders, intervals orders and pseudo orders.

The paper is organized in different sections: after a brief introduction (section 1), section 2 makes a review of PROMETHEE Methods in their original formulation. In section 3, semiorder, interval order and pseudo order preference structures are analyzed as New Preference Structures to be introduced in PROMETHEE Methods. Section 4 describes the running of the PROMETHEE I and II under the NPS. An illustrative example is proposed in section 5, in order to compare the results obtained with the new preference structures and the original ones. Some general conclusions are put forward in section 6 and references are listed in section 7.

2. PROMETHEE methods: A brief review

This section briefly outlines the PROMETHEE I and II Methods before considering their running under the New Preference Structures (NPS). A complete description of these methods and their extension is given in (Brans, Mareschal, Vincke, 1984) (Fernández, 1991).

The main objective of researches has been trying to give a methodology to solve Multicriteria problems that are as simple as possible and easily understood by the decision maker. For that reason, among the Outranking Relations Methods, the PROMETHEE are considered as a good representative.

2.1 *Valued Outranking Relation. Unicriterion preference function. Multicriteria Preference Index*

Let us call A the set of all the possible alternatives to be rank, and $g_1, g_2, \ldots, g_k$ the k criteria that have been selected. The PROMETHEE Methods first build a value outranking relation on A. This relation is then used to obtain a partial (PROMETHEE I) or a complete preorder (PROMETHEE II) on A (Brans, Vincke, 1985).

In each criterion g_i and every two alternatives $a, b \in A$, the preference of the decision maker for a over b is expressed through a preference function P_i such that $0 \leq P_i(a, b) \leq 1, \ \forall a, b \in A$.

$P_i(a, b)$ may be considered as the intensity of preference of a over b, it is equal to 0 in case of no preference of a over b or indifference between a and b, it is equal to 1 in case of strict preference of a over b, for the particular criterion g_i.

In practice, $P_i(a, b)$ is often a function of the deviation between the criteria values $d = g_i(a) - g_i(b), \quad P_i(a, b) = P_i(d)$.

P_i is a non-decreasing function of d and is equal to 0 for negatives values of d.

In the formulation of the generalized criteria, these methods consider six possible types, although they are neither exhaustive nor restricted (Brans, Vincke, 1985). However different extension of the generalized criteria can be seen in Fernández, Escribano, (2006).

Let us suppose that a preference function P_i and a weight w_i have been specified for each criterion g_i ($i = 1, 2, \ldots, k$).

The Multicriteria preference index π is defined as: $\pi(a, b) = \frac{\sum w_i P_i(a,b)}{\sum w_i} \quad \forall \, a, b \in A$

$\pi(a, b)$ represents the intensity of preference of the decision maker of a over b, when considering simultaneously all the criteria. This index defines a valued outranking relation on A.

2.2 *Flows and Rankings*

PROMETHEE defines for each $a \in A$:

- Its leaving flow or positive flow: $\varphi^+(a) = \sum_{b \in A} \pi(a, b)$, giving the *outranking character of a,*
- Its entering glow or negative flow: $\varphi^-(a) = \sum_{b \in A} \pi(b, a)$, giving the *outranked character of a,*
- Its net flow: $\varphi(a) = \varphi^+(a) - \varphi^-(a)$, it is a balance between the positive and negative flows.

The higher the leaving flow the lower the entering flows, the better the alternative. The net flow is given a complete preorder defined as:

$$a \text{ outranks } b \text{ iff } \varphi(a) > \varphi(b)$$
$$a \text{ is indifferent to } b \text{ iff } \varphi(a) = \varphi(b).$$

It can be noticed that the PROMETHEE I offers the decision maker a partial relation that is a partial preorder on the set of alternatives (including possible incomparabilities) to solve the decision problem. On the other hand the PROMETHEE II offers a complete ranking that is a total preorder. In this way, it removes the incomparabilities and seems to be more efficient even though the information used is more disputable and is not completely sure (Brans, Mareschal, 1989, 1990).

However, the methods such as they had been developed and which characteristics had been previously outline, do not get to satisfy the increasing necessity of a more realistic modelization of the decision maker's preferences.

On account of that reason an extension of the PROMETHEE Methods is presented, based on the adoption of New Preference Structures (NPS) not utilized in the field of the outranking relations up till now, namely: semiorders, interval orders and pseudo orders. Prior definition and characterization, those structures are incorporated in every one of the steps that include the PROMETHEE Methods. In this way the analysis is notably enriched, providing a great interactivity and finally, the ranking problem can be solved in a more realistic way, obtaining a partial semiorder, a partial intervals order or a partial pseudo order under the PROMEHTEE I, or alternately, the same rankings but total one's under the PROMETHEE II(Fernández, 1993, 1997).

3. New Preference Structures (NPS): Semiorder, Intervals Order and Pseudo Order

The most recent approaches in the field of Multiple Discrete Decision Making (MDM) are aimed to the development of decision processes that take into account of preference structures even though more complex than the traditional one's, that notably enrich the modelization step.

For that reason it is very interesting to analyze in what extent the consideration of Semiorder, Intervals Order and Pseudo Order Preference Structures within the PROMETHEE Methods improves the decision making process in its entirety in a very important way (Chandon, Vincke, 1981) (Vincke, 1981a, 1981b, 1988).

Having analyzed exhaustively the mentioned preference structures (Roubens,Vincke, 1988) it can be said that they give more flexibility, amplitude and certainty to the preference formulations, as they tend to abandon the Complete Transitive Comparability Axiom of the Preference to replace it by the Partial Comparability Axiom of Preference (Roy, Vincke, 1984). Going from an axiom to other it is possible to introduce in the analysis the Incomparability Relation (R) that is mainly present when: the analyst is not able to discriminate between two alternatives since the information that he has, is too subjective or too incomplete to produce a judgment of Indifference or Strict Preference, the analyst is in a position that no allow himself to determine the decision maker's preferences, since the decision maker may be inaccessible, being either a remote entity or a loose entity with ill-define and/or contradictory preferences, the analyst does not want to discriminate and he prefers to remain removed from the decision process and wait until a later stage when he has more reliable and sure information about the decision maker's preferences (Fernández, 1999).

Another outstanding question is the matter of preference structure transitivity. The Indifference (I), the Strict Preference (P) and the Weak Preference (Q) Relations to be considered in the preference structures are keeping away from the classic theory in the sense that, they are not necessarily transitive (Tversky, 1969).

4. Generalization of the notion of criterion: derivation of the New Preference Structures (NPS)

One of the most important requisites that a Multicriteria method must observe to turn out suitable is to take into account the amplitude of the deviations between the criteria values $[\, d = g_i(a) - g_i(b)]$ (Escribano, Fernández, 2002).

The PROMETHEE Methods consider that question by introducing the notion of quasi-criterion and pseudo-criterion, that is to say, by considering for each criterion some possible extensions. For some of the criteria extension, the intransitivity of Indifference is allowed, for others, it is possible to pass smoothly from Indifference to Strict Preference. They therefore use the notion of criterion. It is important to emphasize that, the main feature of these methods is that each possible extension is very clear and easily understood by the decision maker, which originates a certain natural inclination on the part of him to actively participate during the whole decision process.

The generalization of the notion of criterion gives the idea of intensity of decision maker's preferences on the set of alternatives with regard to them he must select. Therefore, it is of great utility to apply several generalized criteria to the NPS, in order that the process of preferences formulation can gather situations such as incomparability, certain intransitivity and the ambiguity between preference and indifference, being overcome, in this way, the classical notion of criterion (Fernández, 1998).

Six useful shapes have been employed (Brans, Mareschal, Vincke, 1984) and it had been experimentally proved that the six recognized types of generalized criteria are perfectly adapted to the preference structure suggested (NPS) and underlying in the decision problem.

However, some additional types of generalized criteria have been proposed and have been applied to real situations (García, Fernández, Ródenas, 2009). These new preference functions are the result of the combination of two or more of the six existing types. Without any doubt, the possibility to have new preference functions reflecting more precisely the decision maker' preferences, increasing the range of preferences and showing different nuances in their intensity, represents an extraordinary improves of the methodology as a whole. Besides, it allows softening one of the main disadvantages that is attributed to these methods, which does the subjectivity exist at the time to define the preference and or indifference thresholds to be required at any case (Fernández, Escribano, Calvo, 1997).

5. PROMETHEE I and II in front of the new preference structures (NPS)

5.1 The ranking problem of the alternatives

The *enrichment of the preference structures,* first stage of the decision process, takes into account the extension of the notion of criterion and the fixing of parameters with a real economic meaning. The second phase consists on the *enrichment of the dominance relation*. In this second step, the formulation of value outranking relations is considered for the treatment of the problem in a Multicriteria framework and a value outranking graph is obtained by representing the decision maker's preferences (Fernández, 2002).

The ranking problem is approached from the fuzzy outranking graph and this problem belongs to the following phase called *exploitation of the outranking relation for decision aid* (Brans, Mareschal, 1989). The task consists of the utilization of the valued outranking graph to build a total semiorder, a total intervals order, or a total pseudo-order on the A set, or else the same rankings but partial one's if the total result to be too excessive themselves (Vincke, 1980a, 1980b).

In the application of the PROMETHEE I and II Methods to solve the ranking problem, a set of good alternatives can be obtained from such ranking and the proceeding to solve the choice problem.

It is in the preset decision context where the PROMETHEE Methods will be used to solve the ranking problem of the alternatives but facing to the New Preference Structures (Roubens, 1989).

Both, PROMETHEE I and II, will be applied to solve the problem of ranking the alternatives but considering such ranking must answer to some of the following possibilities:

a. Partial and total Semiorder.
b. Partial and total Intervals Order.
c. Partial and total Pseudo-Order.

In this way, the PROMETHEE offers a partial relation on A, including feasible incomparabilities, whereas the PROMETHEE II offers a complete ranking.

5.2 PROMETHEE I and the partial rankings[1]

5.2.1 Partial Semiorder of the PROMETHEE I

Two semiorders are deduced from the positive and negative flows. Moreover, a parameter K called threshold of Outranking Indifference is introduced, which is defined as follows:

$$K = \left(\frac{1}{n}\right) \sum_{i=1}^{n} \left(\frac{q_i}{Max\ g_i}\right),$$

where:

q_i: is the indifference threshold of the generalized criterion assigned to the i criterion, $i = 1 \dots n$,

[1] The analysis of the partial intervals order is not presented in this context because the hypothesis that the threshold of Outranking Indifference introduced, is no longer a constant but a function seem to be very unrealistic.

$Max\ g_i$: is the greatest possible value that can be acquired by whichever alternative under the i criterion,

N: is the number of generalized criteria that require the determination of an indifference threshold.

It is important to notice that it would be always $n > 0$, since the calculation of K could not make any sense for $n \leq 0$. Moreover, it is necessary that $Max\ g_i \neq 0$ least by one of the i criteria.

In this way, the threshold of Outranking Indifference is defined as the arithmetic mean of the values that expresses the importance in relative terms of each g_i criterion.

The K parameter, which fixing process requires the interactivity between the decision maker and the analyst, indicates the greatest value below which an indifference feeling exists between the outranking character of the alternatives, in such a way that the outranking power of a is considered indifferent to the outranking power of b insofar as it does not outrank the K threshold (Fernández, 1993).

Having introduced this new element, the two semiorders will be (P^+, I^+) and (P^-, I^-)

$$
\begin{aligned}
&a\,P^+b \Leftrightarrow \varphi^+(a) > \varphi^+(b) + K \\
&a\,I^+b \Leftrightarrow -K \leq \varphi^+(a) - \varphi^+(b) \leq K \\
&\Leftrightarrow \begin{cases} \varphi^+(a) \leq \varphi^+(b) + K \\ \varphi^+(b) \leq \varphi^+(a) + K \end{cases} \\
&a\,P^-b \Leftrightarrow \varphi^-(a) + K < \varphi^-(b) \\
&a\,I^-b \Leftrightarrow -K \leq \varphi^-(b) - \varphi^-(a) \leq K \\
&\Leftrightarrow \begin{cases} \varphi^-(b) \leq \varphi^-(a) + K \\ \varphi^-(a) \leq \varphi^-(b) + K \end{cases}
\end{aligned}
$$

The partial relation of the PROMETHEE I results of the intersection of these two semiorders:

$$
\begin{array}{lll}
aP^Ib \;\; if & \begin{cases} aP^+b \;\; and \;\; aP^-b & \begin{cases} \varphi^+(a) > \varphi^+(b) + K, and \\ \varphi^-(a) + K < \varphi^-(b) \end{cases} \\ \\ aP^+b \;\; and \;\; aI^-b & \begin{cases} \varphi^+(a) > \varphi^+(b) + K, and \\ \varphi^-(a) + K \geq \varphi^-(b) \end{cases} \\ \\ aI^+b \;\; and \;\; aP^-b & \begin{cases} \varphi^+(a) \leq \varphi^+(b) + K, and \\ \varphi^-(a) + K < \varphi^-(b) \end{cases} \end{cases} & \\
aI^Ib \;\; if & aI^+b \; and \;\; aI^-b & \begin{cases} \varphi^+(a) \leq \varphi^+(b) + K, and \\ \varphi^-(a) + K \geq \varphi^-(b) \end{cases} \\
aR^Ib & otherwise & \begin{cases} \varphi^+(a) > \varphi^+(b) + K, and \\ \varphi^-(a) + K > \varphi^-(b) \end{cases} \\
 & & \begin{cases} \varphi^+(a) < \varphi^+(b) + K, and \\ \varphi^-(a) + K < \varphi^-(b) \end{cases}
\end{array}
$$

Then, in order that an alternative a be positively preferred to another b, it is necessary that the outranking power of a gathered by the positive or output flows be greater than the outranking power of b plus the threshold of Outranking Indifference. Whereas to be negatively preferred, it is necessary that the negative or input flows of a plus the respective threshold of Outranking Indifference be lower than the negative flows of b. It is to say that the weakness of a must be lower than the weakness of b, taking into account that situation where both of them have the same outranking power (Indifference of Outranking).

The conclusion that can be obtained from the partial relation given by the PROMETHEE I method and resulting on the intersection of the considered semiorders, are the following:

1. aP^Ib: *a outranks b*. In this case it can be clearly proved that the greatest outranking power of a is associated to a lowest weakness of a. it is to say that for the (aP^+b and aP^-b) combination, the positive flow is confirmed by the negative flow, both flows are coherent and, in spite of including an area of outranking indifference, the supplied information is sure.

I does not result so clear for whichever of the other relations where the indifference appears, that is in (aP^+b and aI^-b) and in (aI^+b and aP^-b), the task becomes more difficult and it turns to a bit more subjective due to the necessity of introducing the threshold of Outranking Indifference.

2. aI^Ib: *a is indifferent to b*. It can be observed that the positive flow of a maintains coherence with the positive flow of b, since the lowest outranking power of a would be corresponded with a low outranking power of b that is reinforced by an indifferent feeling produced by K, The negative flow of a

represents a low weakness, but being reinforced by a feeling of outranking indifference, it is corresponded with a low weakness of b. This means that, if the area of outranking indifference would not exist both flows might be exactly equals.

3. *aR^Ib: a is incomparable to b.* In this case a higher outranking power of an alternative is corresponded with a lower weakness of the other. This usually happens when a is good on a set of criteria on which b is weak, it is to say, that it has a lower outranking power and reciprocally, b has a higher outranking power under a set of criteria on which a is weak. It seems quite natural and normal to consider a and b as incomparable. Here, it can be seen that there is not any correspondence between the positive and the negative flows. The method does not solve mathematically the problem and it is up to the decision maker to decide which alternative is, in his opinion, the best.

5.2.2 Partial Pseudo-Order of the PROMETHEE I. The pseudo-criteria supporting the Pseudo-Order Preference Structure

The preference structure used for supporting the most general type of criterion, that is to say a pseudo criterion, is a too much complex structure named pseudo order (Roy, Vincke, 1987).

There are decision situations in which the decision maker hesitates between Strict Preference and Indifference, because of the imprecision, irresolution and uncertainty of the information he has or since the criteria evaluations do not constitute a clear representation of the reality, taking place the appearance of the so-called Weak Preference$(Q = P \cup I)$ (Roy, 1977). When the modelization of the preference includes this relation of Weak Preference, it is necessary to consider not only an indifference threshold but a preference threshold as well. The pseudo criteria are used in this stage, as they have the capacity to discriminate between Indifference, Weak Preference and Strict Preference, based on the differences between the evaluations received under each criterion, that is to say [$d = g_i(a) - g_i(b)$].

A preference structure defined as a pseudo-order perfectly supports to a pseudo criterion, having to carry out some previous conditions, such as it is mentioned in a theorem proposed by Vincke (Roy, Vincke, 1984).

The essential aim of this type of ranking consists on the obtaining of a partial pseudo-order form the positive and the negative outranking flows. With this aim in view, a new threshold is additionally incorporated to the already defined Outranking Indifference *(K)* one, it is the threshold of Outranking Preference *(F)*.

The threshold of Outranking Preference is defined as follows:

$$F = \left(\frac{1}{n}\right) \sum_{i=1}^{n} \left(\frac{p_i}{Max\ g_i}\right)$$

where:

p_i: is the preference threshold of the generalized criterion assigned to the i criterion, $i = 1 \dots n$,

Max g_i: is the greatest possible value that can be acquired by whichever alternative under the i criterion,

n: is the number of generalized criteria that require the determination of a preference threshold.

Just as in the semiorder structure, it is important to notice that it would be always $n > 0$ since the calculation of K and F could not make any sense for $n \leq 0$. Moreover, it is necessary that $Max\ g_i \neq 0$ at least by one of the i criteria.

Therefore, the threshold of Outranking Preference is defined as the arithmetic mean of the values that represent the importance in relative terms of each threshold p_i with to the maximum values of each g_i criterion.

With the aim of calculating the threshold of Outranking Preference it is supposed that both p_i and g_i exist for each criterion and, of course, $p_i > g_i, \forall i$. In that way, there is no doubt that $F > K$ as it is proved by the two pseudo orders formulations.

The F parameter indicates the lower value above which it exits a preference feeling between the outranking characters of the alternative, it is to say, that although the alternatives have a strong outranking power themselves, there is a preference from some of them over the other. This threshold is extended beyond the area where an indifference feeling exists between the outranking characters of the alternatives.

The two pseudo orders will be (P^+, I^+, Q^+) *and* (P^-, I^-, Q^-):

$$aP^+b \quad \Leftrightarrow \quad \varphi^+(a) > \varphi^+(b) + F$$
$$aI^+b \quad \Leftrightarrow \quad -K \leq \varphi^+(a) - \varphi^+(b) \leq K$$

$$\Leftrightarrow \begin{cases} \varphi^+(a) \le \varphi^+(b) + K \\ \varphi^+(b) \le \varphi^+(a) + K \end{cases}$$
$$aQ^+b \Leftrightarrow K \le \varphi^+(a) - \varphi^+(b) \le F$$
$$\Leftrightarrow \begin{cases} \varphi^+(a) \le \varphi^+(b) + F \\ \varphi^+(a) > \varphi^+(b) + K \end{cases}.$$
$$aP^-b \Leftrightarrow \varphi^-(a) + F < \varphi^-(b)$$
$$aI^-b \Leftrightarrow -K \le \varphi^-(b) - \varphi^-(a) \le K$$
$$\Leftrightarrow \begin{cases} \varphi^-(b) \le \varphi^-(a) + K \\ \varphi^-(a) \le \varphi^-(b) + K \end{cases}$$
$$aQ^-b \Leftrightarrow K < \varphi^-(b) - \varphi^-(a) \le F$$
$$\Leftrightarrow \begin{cases} \varphi^-(b) > \varphi^-(a) + K \\ \varphi^-(b) \le \varphi^-(a) + F \end{cases}$$

The partial relation of the PROMETHEE I results from the intersection of the two previous pseudo orders:

$$aP^Ib \;\; if \begin{cases} aP^+b \;\; and \;\; aP^-b & \begin{cases} \varphi^+(a) > \varphi^+(b) + F, and \\ \varphi^-(a) + F < \varphi^-(b) \end{cases} \\ aP^+b \;\; and \;\; aI^-b & \begin{cases} \varphi^+(a) > \varphi^+(b) + F, and \\ \varphi^-(a) + K \ge \varphi^-(b) \end{cases} \\ aP^+b \;\; and \;\; aQ^-b & \begin{cases} \varphi^+(a) > \varphi^+(b) + F, and \\ K < \varphi^-(b) - \varphi^-(a) \le F \end{cases} \\ aI^+b \;\; and \;\; aP^-b & \begin{cases} \varphi^+(a) \le \varphi^+(b) + K, and \\ \varphi^-(a) + F < \varphi^-(b) \end{cases} \\ aQ^+b \;\; and \;\; aP^-b & \begin{cases} K < \varphi^+(a) - \varphi^+(b) \le F, and \\ \varphi^-(a) + F < \varphi^-(b) \end{cases} \end{cases}$$

$$aI^Ib \;\; if \qquad aI^+b \; and \; aI^-b \quad \begin{cases} \varphi^+(a) \le \varphi^+(b) + K, and \\ \varphi^-(a) + K \ge \varphi^-(b) \end{cases}$$

$$aQ^Ib \;\; if \begin{cases} aQ^+b \;\; and \;\; aQ^-b & \begin{cases} K < \varphi^+(a) - \varphi^+(b) \le F, and \\ K < \varphi^-(b) - \varphi^-(a) \le F \end{cases} \\ aQ^+b \;\; and \;\; aI^-b & \begin{cases} K < \varphi^+(a) - \varphi^+(b) \le F, and \\ \varphi^-(a) + K \ge \varphi^-(b) \end{cases} \\ aI^+b \;\; and \;\; aQ^-b & \begin{cases} \varphi^+(a) \le \varphi^+(b) + K, and \\ K < \varphi^-(b) - \varphi^-(a) \le F \end{cases} \end{cases}$$
$$aR^Ib \qquad otherwise$$

It can be noticed that the number of possible relations is considerably extended on incorporation the Weak Preference relation and on defining two new thresholds that result from the generalization of the preference and indifference thresholds of the original preference structure.

When making binary comparisons, the PROMETHEE I establishes a pseudo-order on the *A* set. In that way, they appear four well-differentiated situations, namely:

1. aP^Ib: *a outranks b*: This is a situation of Strict Preference and characterized by a wide correspondence between positive and negative flows. That is to say, the higher outranking power of one alternative is corresponding with its lower weakness. This is the surest information that can be collected in the analysis.

2. aI^Ib: *a is indifferent to b:* here, there is also correspondence, but it appears between the positive and negative flows themselves of each alternative.

3. *aQ^Ib: a weakly outranks b*: This new situation of Weak Preference appears when establishing a pseudo-order between the alternatives. It is required the consideration of the two threshold, named as the threshold of Outranking Indifference and the threshold of Outranking Preference. The outranking feeling of one alternative is not too strong and there is doubt between preference and indifference, for that reason it is reasonable to consider it as a feeling of weak outranking. In this case, the flows are coherent, as well. By this means for instance, a higher outranking power of one alternative is corresponding to a lower weakness, having previously incorporated the threshold of Outranking Indifference.

4. *aR^Ib: a is incomparable to b:* Finally, there are situation that are not possible to be compared since there is no coherence between the flows. It generally occurs when the strong discriminatory power of one alternative makes it to be good under a set of criteria in which the other alternative is weak, this is to say that the negative flows of the second alternative are lower than the first one, being therefore less weak than.

5.3 The PROMETHEE II and the total rankings

5.3.1 *Total Semiorder of the PROMETHEE II*

When a complete or total semiorder is required (Jamison, Lau, 1973), avoiding whichever incomparability, the net outranking flows may be considered:

$$\varphi(a) = \varphi^+(a) - [\varphi^-(a) + K], or\ else$$
$$\varphi(a) = \varphi^+(a) - \varphi^-(a) - K$$

where: $\varphi^+(a)$ is the outranking power of a,
$\varphi^-(a)$ is the weakness degree of a,
K is the indifference feeling (threshold of Outranking Indifference).

A net outranking flow takes place of a balance among the several intensity expressions of the preferences that there are in the indices of preference and behind the flows. The higher the value takes by the net flow, the better the alternative in question.

The complete ranking is defined by *(P^{II},I^{II}):*

$$aP^{II}b \quad \Rightarrow \quad \varphi(a) > \varphi(b)$$
$$aI^{II}b \quad \Rightarrow \quad \varphi(a) = \varphi(b)$$

but remembering that the threshold of Outranking Indifference K takes part in the net flows determination.

At a glance, it can be proved that it is easier for the decision maker to solve the problem using a complete semiorder, in such a way that all the alternatives can be compared. However, a partial semiorder offers more realistic information since the data in relation to the incomparabilities can frequently be very useful for the decision to be made (Luce, 1956).

5.3.2 *The Total Pseudo Order of the PROMETHEE II*

For the case of the complete or total pseudo order of the PROMETHEE II the net outranking flows must be previously considered:

$$\varphi(a) = [(\varphi^+(a) + F) - (\varphi^-(a) + K)]$$

where the first term of the difference represents the outranking character plus the gap of preference and the second, the outranked character plus the gap of indifference.

The complete ranking under the pseudo order structure would be defined by *(P^{II}, I^{II}, Q^{II}):*

$$aP^{II}b \quad \Rightarrow \quad \varphi(a) > \varphi(b) + F$$
$$aI^{II}b \quad \Rightarrow \quad -K \leq \varphi(a) - \varphi(b) \leq K$$
$$aQ^{II}b \quad \Rightarrow \quad K < \varphi(a) - \varphi(b) \leq F$$

Here, the incomparabilities do not appear which supposes a loss of information, being very valuable at the moment of making the final decision.

6. Illustrative example: The location of an electric power plant

With the aim of compare the results obtained using the software that supports the PROMETHEE methodology[2] with those coming from the performance of the personally elaborated programs which are considered as an extension of the previous ones (Fernández, 1991) , the application of both methods to a case that appeared in the specialized literature about PROMETHEE has been carefully studied.

The main objective of the comparison is to obtain conclusions, pointing out the advantages and disadvantages of the new analysis that stems from the application of the method, enriched with the proposed changes.

6.1 A short description of the problem

Brans and Mareschal (1989) studied a problem referred to the selection among six alternatives of electric power plants location, considering six criteria as the most important for the decision.

Table 1 shows the evaluations of the six alternatives, the type of generalized criterion respective to each original criterion and its corresponding parameters.

Table 1. The six alternatives and the types of generalized criteria

Criteria / Alternatives	g_1 Minimize	g_2 Maximize	g_3 Minimize	g_4 Minimize	g_5 Minimize	g_6 Maximize
A_1: Italy	80	90	60	5.4	8	5
A_2: Belgium	65	58	20	9.7	1	1
A_3: Germany	83	60	40	7.2	4	7
A_4: Sweden	40	80	100	7.5	7	10
A_5: Austria	52	72	60	2.0	3	8
A_6: France	94	96	70	3.6	5	6
Generalize Criteria Type	II	III	V	IV	I	VI
Parameters:						
q	10	-	5	1	-	-
p	-	30	45	5	-	-
Σ	-	-	-	-	-	5

The evaluation criteria are:

g_1: Manpower for running the plant (in thousands of millions U$D),
g_2: Power (in Megawatt),
g_3: Construction costs (in thousands of millions U$D),
g_4: Annual maintenance costs (in thousands of millions U$D),
g_5: Number of villages to be evacuated,
g_6: Safety level.

The PROMETHEE flows, positive, negative and net of the alternatives appear in *Table 2*. These are the results that will be compared with the ones offered by the new methodology and they constitute the starting point for its development.

Table 2. PROMETHEE Flows

Alternatives	$\phi^+(.)$	Order	$\phi^-(.)$	Order	$\phi(.)$	Order
A_1: Italy	0.222	6	0.366	5	-0.146	6
A_2: Belgium	0.396	2	0.379	6	0.017	2
A_3: Germany	0.247	5	0.336	2	-0.090	5
A_4: Sweden	0.329	3	0.349	3	-0.020	3
A_5: Austria	0.455	1	0.162	1	0.293	1
A_6: France	0.300	4	0.355	4	-0.055	4

[2] A first implementation was realized at the Université Libre de Bruxelles (ULB) at the end of the 1980's. It was named **PROMCALC** and was running under MS-DOS. It was one of the first really interactive software based on outranking methods. At the end of the 1990's **Decision Lab** was developed in a joint venture between the ULB and the Canadian company Visual Decision. New **PROMETHEE-GAIA** software named Visual PROMETHEE is developed by Bertrand Mareschal at ULB.

The PROMETHEE I partial ranking (partial preorder) and its respective graph are:

$A_2\ R^I\ A_1$	$A_3\ R^I\ A_1$	$A_4\ P^I\ A_1$	$A_5\ P^I\ A_1$	$A_6\ P^I\ A_1$
$A_2\ R^I\ A_3$	$A_3\ R^I\ A_4$	$A_4\ P^I\ A_6$	$A_5\ P^I\ A_2$	
$A_2\ R^I\ A_4$	$A_3\ R^I\ A_6$		$A_5\ P^I\ A_3$	
$A_2\ R^I\ A_6$			$A_5\ P^I\ A_4$	
			$A_5\ P^I\ A_6$	

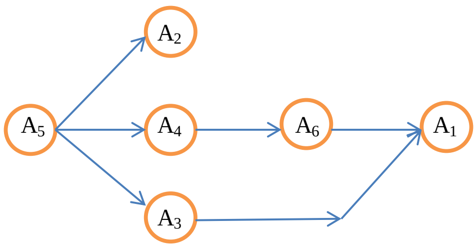

Figure 1.

The PROMETHEE II complete ranking (total preorder) and its respective graph are:

Order	Alternative	Net Flow
1	A_5: Austria	0.293
2	A_2: Belgium	0.017
3	A_4: Sweden	-0.020
4	A_6: France	-0.055
5	A_3: Germany	-0.090
6	A_1 Italy	-0.146

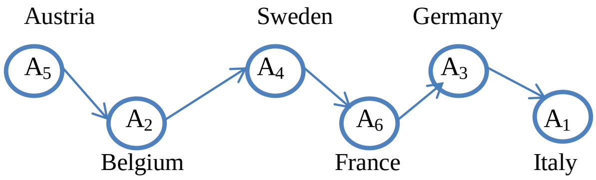

Figure 2.

6.2 Comparative analysis of results: Semiorder Preference Structure

The programs that allow obtaining a partial semiorder or a total one between the alternatives have been enumerated and explained in the research worker previously mentioned (see: Fernández Barberis, Doctoral Thesis).

The partial semiorder between the negative outranking flows or entering flows (SEMIN program) offers a series of interesting results. Then we compare the partial preorder with the partial semiorder as to the negative flows it refers to:

Partial Preorder $\varphi^-(.)$:

$A_1\ P^-\ A_2$	$A_3\ P^-\ A_1$	$A_4\ P^-\ A_1$	$A_5\ P^-\ A_1$	$A_6\ P^-\ A_1$
	$A_3\ P^-\ A_2$	$A_4\ P^-\ A_2$	$A_5\ P^-\ A_2$	$A_6\ P^-\ A_2$
	$A_3\ P^-\ A_4$	$A_4\ P^-\ A_6$	$A_5\ P^-\ A_3$	
	$A_3\ P^-\ A_6$		$A_5\ P^-\ A_4$	
			$A_5\ P^-\ A_6$	

Partial Semiorder $\varphi^-(.)$:

$A_1\ I^-\ A_2$	$A_3\ I^-\ A_1$	$A_4\ I^-\ A_1$	$A_5\ P^-\ A_1$	$A_6\ I^-\ A_1$

$A_3\ I^-\ A_2$	$A_4\ I^-\ A_2$	$A_5\ P^-\ A_2$	$A_6\ I^-\ A_2$
$A_3\ I^-\ A_4$	$A_4\ I^-\ A_6$	$A_5\ P^-\ A_3$	
$A_3\ I^-\ A_6$		$A_5\ P^-\ A_4$	
		$A_5\ P^-\ A_6$	

It must be noticed that in the comparison of the pairs of alternatives the great part of the relations that were previously of Preference (P^-), now, they have been converted into Indifference (I^-). Because of the fact that an alternative is preferred to another with regard to the negative flows, it must be much less weak and not even adding the indifference tolerance represented by K it is able to become weaker than it.

In our example, A_5 is negatively preferred to all and sundry of the remaining ones and although the threshold of Outranking Indifference is added, it is not possible to counter this low degree of weakness that it has, so its outrank character is not influenced by the ranking type (preorder or semiorder).

The most influential element in the changes of preference is the threshold of Outranking Indifference and its value depends, in its turn, on the threshold that have been individually determined for every type of generalized criterion. The higher K, the alternative will be weaker and by the contrary, the smaller K, the negative flow will be nearly modified and the alternative will continue to maintaining, approximately, the same weakness.

The partial semiorder between the positive outranking flows or leaving flows (SEMIP program) gives place to the emerging of a great number of Indifference relations, that is to say, there are changes in the preferences and the preferred alternatives (P^+) are the ones which have a great outrank power, that cannot even be countered by adding the K threshold to the positive flows of the another alternative component of the pair to be compared.

The results of the partial preorder and the partial semiorder referred to the positive flows are the following:

Partial Preorder φ^+ (.):

$A_2\ P^+\ A_1$	$A_3\ P^+\ A_1$	$A_4\ P^+\ A_1$	$A_5\ P^+\ A_1$	$A_6\ P^+\ A_1$
$A_2\ P^+\ A_3$		$A_4\ P^+\ A_3$	$A_5\ P^+\ A_2$	$A_6\ P^+\ A_3$
$A_2\ P^+\ A_4$		$A_4\ P^+\ A_6$	$A_5\ P^+\ A_3$	
$A_2\ P^+\ A_6$			$A_5\ P^+\ A_4$	
			$A_5\ P^+\ A_6$	

Partial Semiorder φ^+ (.):

$A_2\ P^+\ A_1$	$A_3\ I^+\ A_1$	$A_4\ P^+\ A_1$	$A_5\ P^+\ A_1$	$A_6\ I^+\ A_1$
$A_2\ P^+\ A_3$		$A_4\ I^+\ A_3$	$A_5\ P^+\ A_3$	$A_6\ I^+\ A_3$
$A_2\ I^+\ A_4$		$A_4\ I^+\ A_6$	$A_5\ P^+\ A_4$	
$A_2\ I^+\ A_5$			$A_5\ P^+\ A_6$	
$A_2\ P^+\ A_6$				

By going on with a_5, it can be observed that it reduced its own outrank power in front of a_2 which reinforced it, so that in the new ranking both alternatives are indifferent. Similar changes appear between (a_3, a_1), (a_2, a_4), (a_4, a_3), (a_4, a_6), (a_6, a_1) and (a_6, a_3) which demonstrates that the alternatives a_1 and a_3 are the ones which have more reinforced their outrank power since before they were preferred by alternatives with regard to, now, they are indifferent.

Another aspect to be outstanding is that, from now on, in these former partial rankings clearly appear comparisons that seen briefly the Intransitivity of the Preferences, for instance:

$A_2\ I^+\ A_4$,	$A_4\ I^+\ A_3$ but	$A_2\ P^+\ A_3$
$A_5\ I^+\ A_2$,	$A_2\ I^+\ A_4$ but	$A_5\ P^+\ A_4$
$A_1\ I^+\ A_6$,	$A_6\ I^+\ A_4$ but	$A_4\ P^+\ A_1$

not been fulfilled in them the transitivity of the Indifference $(I^2 \not\subset I)$. In other relations it must be noticed that both the Preference and the Indifference are transitive, giving place to situations where $(IP \subset P)$ or $(PI \subset P)$ are not fulfilled, just as the following exemplified:

$A_4\ I^+\ A_2$,	$A_2\ P^+\ A_3$but	$A_4\ I^+\ A_3$
$A_2\ I^+\ A_5$,	$A_5\ P^+\ A_4$but	$A_2\ I^+\ A_4$
$A_4\ I^+\ A_2$,	$A_2\ P^+\ A_6$but	$A_4\ I^+\ A_6$
$A_4\ P^+\ A_1$,	$A_1\ P^+\ A_3$but	$A_4\ I^+\ A_3$.

As can be seen, both partial semiorders allow to the decision-maker to express more freedom his preferences, without to having been submitted to the inflexibility of the traditional scheme and giving to a wide acceptation to the Partial Comparability Axiom.

To be worth to put attention in the fact that in spite of the changes that have been being internally originated in the partial semiorders, if we rank the negative and the positive flows in accordance with theirs absolute values, we would obtain the same results that in the reference method. *Table 3* offers the negative and the positive flows of the alternatives and the rank they have inside of the structure being studied.

Table 3. PROMETHEE Flows (Partial Semiorder)

Alternatives	$\phi^+(.)$	Order	$\phi^-(.)$	Order
A_1: Italy	0.306	6	0.452	5
A_2: Belgium	0.482	2	0.465	6
A_3: Germany	0.333	5	0.422	2
A_4: Sweden	0.415	3	0.435	3
A_5: Austria	0.541	1	0.248	1
A_6: France	0.386	4	0.441	4

The PROMETHEE I partial relation (Partial Semiorder) results from the intersection of the two previous semiorders but before going pass to the total semiorder we need a series of additional calculations (SEMIF and SEMIG programs).

In fact, due to the appearance of intransitive relations it is necessary to carry out a new ranking among the pairs of alternatives, this is, to obtain the intersection between the result that emerges from the comparison of the pair *(a, b)* and the one which emerges from the pair *(b, a)* by all and sundry of the alternatives in question.

The three situations that can appear are: Preference, Indifference and Incomparability. In the present example, the partial resultant semiorder is the following:

$A_2\ P^+A_1$ and	$A_2\ I^-A_1$	$\Rightarrow$	$A_2\ P^IA_1$
$A_2\ P^+A_3$ and	$A_3\ I^-A_2$	$\Rightarrow$	$A_2\ P^IA_3$
$A_2\ I^+A_4$ and	$A_2\ I^-A_4$	$\Rightarrow$	$A_2\ I^IA_4$
$A_2\ P^+A_6$ and	$A_2\ I^-A_6$	$\Rightarrow$	$A_2\ P^IA_6$
$A_3\ I^+A_1$ and	$A_3\ I^-A_1$	$\Rightarrow$	$A_3\ I^IA_1$
$A_4\ P^+A_1$ and	$A_4\ I^-A_1$	$\Rightarrow$	$A_4\ P^IA_1$
$A_4\ I^+A_3$ and	$A_4\ I^-A_3$	$\Rightarrow$	$A_4\ I^IA_3$
$A_4\ I^+A_6$ and	$A_4\ I^-A_6$	$\Rightarrow$	$A_4\ I^IA_6$
$A_5\ P^+A_1$ and	$A_5\ P^-A_1$	$\Rightarrow$	$A_5\ P^IA_1$
$A_5\ I^+A_2$ and	$A_5\ P^-A_2$	$\Rightarrow$	$A_5\ P^IA_2$
$A_5\ P^+A_3$ and	$A_5\ P^-A_3$	$\Rightarrow$	$A_5\ P^IA_3$
$A_5\ P^+A_4$ and	$A_5\ P^-A_4$	$\Rightarrow$	$A_5\ P^IA_4$
$A_5\ P^+A_6$ and	$A_5\ P^-A_6$	$\Rightarrow$	$A_5\ P^IA_6$
$A_6\ I^+A_1$ and	$A_6\ I^-A_1$	$\Rightarrow$	$A_6\ I^IA_1$
$A_6\ I^+A_3$ and	$A_6\ I^-A_3$	$\Rightarrow$	$A_6\ I^IA_3$

These latter outcomes make the way to the graphic representation of the PROMETHEE I partial semiorder and subsequent obtaining of the PROMETHEE II total semiorder.

The changes undergone in the relations when passing from the partial preorder to the partial semiorder are the following:

Partial Preorder	Partial Semiorder
$A_2R^IA_1$	$A_2\ P^IA_1$
$A_2R^IA_3$	$A_2P^IA_3$
$A_2R^IA_4$	$A_2\ I^IA_4$
$A_2R^IA_6$	$A_2P^IA_6$
$A_3P^IA_1$	$A_3\ I^IA_1$
$A_3R^IA_4$	$A_3\ I^IA_4$
$A_3R^IA_6$	$A_3\ I^IA_6$
$A_4\ P^IA_6$	$A_4\ I^IA_6$
$A_6\ P^IA_1$	$A_6\ I^IA_1$

and the graph corresponding to the partial semiorder is:

Partial Semiorder

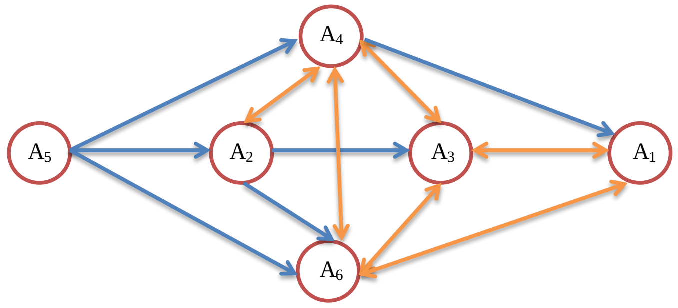

Figure 3.

The analysis is being enriched insofar as it allows to study in depth the preferences that the decision-maker revels at the time of comparing the alternatives by pairs getting, besides, a larger interaction and an active participation of him during the whole decision process.

6.2.1 *Conclusion in relation with the partial ranking*

From the comparison of the partial ranking results obtained by having use of Decision Lab and the SEMIG programs, offered in the preceding paragraph, we can conclude that:

1. The changes in the preferences appear in the weakest alternatives or in those alternatives with a smaller outrank power than others.
2. The incorporation of the Threshold *K* involves the appearance of a large number of Indifference relations. In the present example where we extend the new methodology, almost none of the alternatives are strong enough as to pass to the Preference area outranking *K* in any cases.
3. The intransitivity of the preferences (Preference and Indifference) requires new graphs by representing the partial semiorder.
4. The observation of both the partial resultant ranking and its graphic representation confirms what the theoretical proposals express. The decision-maker can formulate more openly his preferences and he even has the freedom of expressing those feelings of indecision or uncertainty without having to be compulsory adjusted to the relations, up till then, accepted.
5. Another important modification that can be pointed out is that the change in the preferences takes place with regard to the Incomparabilities. This means that the alternatives are not so incompatible each other, since the incorporation of the threshold *K* turned the Incomparability relations into Indifference or Preference. Both the reference and the modified methods allow the existence of Incomparabilities but in the last case the positive and the negative flows have to be strongly different each other for the Incomparabilities appearance.

6.2.2 *Conclusion in relation with the complete ranking*

The total semiorder, this is to say, the PROMETHEE II complete ranking (SEMTOT program) offers net flows which are different in absolute value from the ones of the reference method but at the time of ranking them, they produced the same final result.

Table 4 shows the net flows and the rank of every alternative in accordance with these flows:

Table 4. Net Flows

Alternatives	$\phi(.)$	Order
A_1: Italy	-0.232	6
A_2: Belgium	-0.069	2
A_3: Germany	-0.175	5
A_4: Sweden	-0.106	3
A_5: Austria	0.207	1
A_6: France	-0.141	4

Same changes are internally produced in the calculations of the net flows. In the new situation these modifications are reflected in the NPS and they lead to the last step in which at first, we rank the net flows and then the alternatives in accordance with their respective net flows (PROME2 program).

In that way, the PROMETHEE II complete ranking (Total Semiorder) is the following:

Order	Alternative	Net Flow
1	A_5: Austria	0.207
2	A_2: Belgium	-0.069
3	A_4: Sweden	-0.106
4	A_6: France	-0.141
5	A_3: Germany	-0.175
6	A_1: Italy	-0.232

The graphical representation of the total semiorder is exactly the same that the total preorder and at a glance it can be remarked that the Incomparabilities elimination notably makes easy the decision-maker work but it implicates the loss of useful information at the time of having to take the final decision[3].

6.3 Comparative analysis of results: Pseudo-Order Preference Structure

The programs which allow the obtaining of a total pseudo order or a partial one among the alternatives are enumerated and explained in the research work yet referred (Fernández Barberis, 1991)[4].

A new element to be considered is the threshold of Outranking Preference which actively contributes in the calculations to be necessary in the following steps (FLUJOSNP and FLUJOSPP programs).

The ranking between the negative outranking flows or entering flows but in correspondence with a partial pseudo-order between these floes is a new feature if we compare it with the reference method, due to the preference structure here considered is too much complex and though it, the decision-maker can formulate his preference much openly, having access to hew situations that escape from the traditional of Strict Preference and Indifference and giving place to the appearance of the named Weak Preference.

The first interesting results appear when obtaining the partial pseudo-order between the negative outranking flows or entering flows (PSEUDON program).

If we make a comparison with regard to the reference method we can see that the Preference relations in their entirety are transformed into Indifference (I^-) or Weak Preference (Q^-) due to the repercussions that the incorporation of both thresholds F and K has to, as it can be proved by analyzing the following results:

Partial Pseudo-order $\varphi^-(.)$:

$A_1\ I^-\ A_2$	$A_2\ I^-\ A_3$	$A_3\ I^-\ A_4$	$A_4\ I^-\ A_6$	$A_5\ Q^-\ A_1$
$A_1\ I^-\ A_3$	$A_2\ I^-\ A_4$	$A_3\ I^-\ A_6$		$A_5\ Q^-\ A_2$
$A_1\ I^-\ A_4$	$A_2\ I^-\ A_6$			$A_5\ Q^-\ A_3$
$A_1\ I^-\ A_6$				$A_5\ Q^-\ A_4$
				$A_5\ Q^-\ A_6$

In this way, alternatives visibly preferred are not so whether the decision-maker has the opportunity of freedom expresses his feelings in those cases in which he demonstrates uncertainty to be decided by a Preference in strict sense. This is very important at the time of modeling his preference although the structure that is used as support, be too much complex.

If we compare the partial pseudo-order with the partial semiorder, we can clearly observe how those Strict Preference situations change into Weak Preference when introducing the threshold F. This means that in order to continue keeping up the Preference character in strict sense, the alternatives must have the smallest weakness.

By analogy, we obtain the partial pseudo-order between the positive outranking flows or leaving flows (PSEUDOP program), giving the following results:

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[3] The method of interval orders is not developed because it produces the same results that the semiorder one, with the only difference that the former works with variable thresholds.

[4] Fernández Barberis, G.M. (1991): Extensión a los Métodos PROMETHEE de Nuevas Estructuras de Preferencia para la Toma de Decisiones Multicriterio Discretas. Doctoral Thesis.

Partial Pseudo-order φ^+ (.):

$A_1\ I^+\ A_3$	$A_2\ Q^+\ A_1$	$A_3\ I^+\ A_4$	$A_4\ Q^+\ A_1$	$A_5\ Q^+\ A_1$
$A_1\ I^+\ A_6$	$A_2\ Q^+\ A_3$	$A_3\ I^+\ A_6$	$A_4\ I^+\ A_6$	$A_5\ Q^+\ A_3$
	$A_2\ I^+\ A_4$			$A_5\ Q^+\ A_4$
	$A_2\ I^+\ A_5$			$A_5\ Q^+\ A_6$
	$A_2\ Q^+\ A_6$			

Just as it can be observed and equal for the negative flows, the changes in the preferences are inclined to the appearance of Weak Preference situations, reflecting that no alternative has to an outranking power sufficiently important so as to be strictly preferred to any other and an extensive range of possibilities is opened to the decision-maker at the moment of formulating his preferences.

In both positive and negative pseudo-orders it is very clear that the Preference relations gibe up being transitive and specially when incorporating the Weak Preference to the already known of Strict Preference, Indifference and Incomparability, the intransitivity occur more frequently. In this way, for instance:

$A_1\ I^+ A_3$,	$A_3\ I^+\ A_4$ but	$A_4\ Q^+\ A_1$
$A_4\ I^+ A_6$,	$A_6\ I^+\ A_1$ but	$A_4\ Q^+\ A_1$
$A_2\ I^+ A_4$,	$A_4\ I^+\ A_6$ but	$A_2\ Q^+\ A_6$

these relations do not observe the transitivity of Indifference. In such cases both the Weak Preference and the Indifference are transitive, appearing situations as the next ones:

$A_2\ I^+ A_5$,	$A_5\ Q^+\ A_4$ but	$A_2\ I^+\ A_4$
$A_4\ Q^+ A_1$,	$A_1\ I^+\ A_3$ but	$A_4\ I^+\ A_3$
$A_2\ Q^+ A_3$,	$A_3\ I^+\ A_4$ but	$A_2\ I^+\ A_4$.

The PROMETHEE I partial relation (Partial Pseudo Order) stems from the intersection of two pseudo orders (PSEUDOFI program). In our example, the partial pseudo-order is the following:

$A_1\ I^I\ A_3$	$A_2\ Q^I\ A_1$	$A_3\ I^I\ A_4$	$A_4\ Q^I\ A_1$	$A_5\ Q^I\ A_1$
$A_1\ I^I\ A_6$	$A_2\ Q^I\ A_3$	$A_3\ I^I\ A_6$	$A_4\ I^I\ A_6$	$A_5\ Q^I\ A_2$
	$A_2\ I^I\ A_4$			$A_5\ Q^I\ A_3$
	$A_2\ Q^I\ A_6$			$A_5\ Q^I\ A_4$
				$A_5\ Q^I\ A_6$

the great prevalence of Preference situation (Weak in this case) is due to that in every circuit where there are both Preference and Indifference relations is obvious the higher power of the former ones having these, a strict or a weak character.

This step is previous to the obtaining of the named global pseudo order (PSEUDOG program) which results lead to the graphical representation of the preference structure to be modelized.

The relations that have undergone changes by passing from the partial preorder to the partial pseudo order are the following:

Partial Preorder	Partial Pseudo Order
$A_2R^IA_1$	$A_2\ Q^IA_1$
$A_2R^IA_3$	$A_2Q^IA_3$
$A_2R^IA_4$	$A_2\ I^IA_4$
$A_2R^IA_6$	$A_2Q^IA_6$
$A_3P^IA_1$	$A_3\ I^IA_1$
$A_3R^IA_4$	$A_3\ I^IA_4$
$A_3R^IA_6$	$A_3\ I^IA_6$
$A_4\ P^IA_1$	$A_4\ Q^IA_1$
$A_4\ P^IA_6$	$A_4\ I^IA_6$
$A_5\ P^IA_1$	$A_5\ Q^IA_1$
$A_5\ P^IA_2$	$A_5\ Q^IA_2$
$A_5\ P^IA_3$	$A_5\ Q^IA_3$
$A_5\ P^IA_4$	$A_5\ Q^IA_4$
$A_5\ P^IA_6$	$A_5\ Q^IA_6$
$A_6\ P^IA_1$	$A_6\ I^IA_1$

and the graph of the partial pseudo order is:

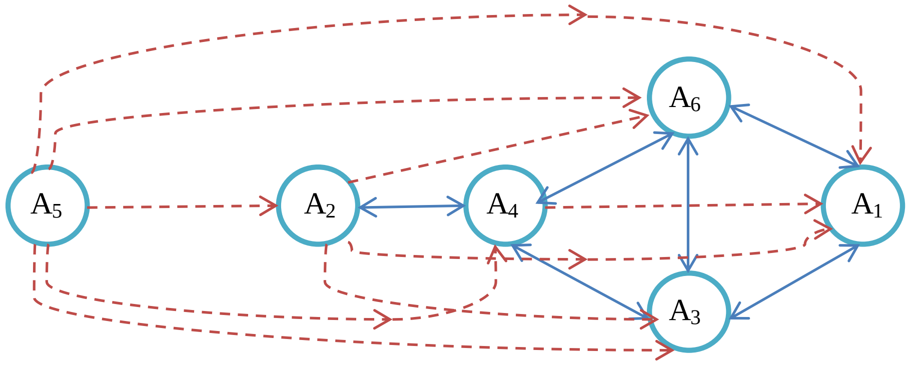

Figure 4.

A characteristic to be outstanding is that the graphic representation of the Weak Preference relations is made with discontinuous lines in order to be different from the rest.

6.3.1 *Conclusion in relation with the partial ranking*

The comparison of the partial ranking results obtained by using the Decision Lab and the PSEUDOG and offered in the previous lines, allows taking out the following conclusions:

1. Practically, all the relations have undergone changes in the preferences.
2. The incorporation of the threshold of Outranking Preference involves the appearance of a large number of Weak Preference relations which indicates us that none of the alternatives is strong enough to pass to the Strict Preference area, outranking the thresholds *K* and *F*.
3. The intransitivity of the preferences (Strict Preference, Indifference and Weak Preference) requires new graphs for the partial pseudo order representation.
4. The observation of both the graphic representation and the results produced by the partial pseudo order allows to confirm those formulations about the range of possibilities that the decision-maker has for freedom express his preferences, just like he can be an active part during the whole decision making process.
5. Finally, the analysis does not exclude the appearance of Incomparabilities even though they are only present when the alternatives to be compared show a strong disparity between their flows, in such a way that they cannot be included in some of the preference situations previously considered.

6.3.2 *Conclusion in relation with the complete ranking*

The total pseudo order, this is to say, the PROMETHEE II complete ranking (PSEUDTOT program) offers the following results:

Table 5. Net Flows

Alternatives	$\phi(.)$	Order
A_1: Italy	0.245	6
A_2: Belgium	0.408	2
A_3: Germany	0.302	5
A_4: Sweden	0.371	3
A_5: Austria	0.684	1
A_6: France	0.336	4

In spite of the internal changes that have been observed in the preferences and also in the absolute values of the flows, the final ranking of the alternatives has not been modified.

Once the net flows are obtained, the alternatives are ranked in accordance with such flows (PROME2P program) so, arriving at the last step of the proposed methodology. Therefore, the PROMETHEE II total pseudo order is then displayed:

Order	Alternative	Net Flow
1	A_5: Austria	0.684
2	A_2: Belgium	0.408
3	A_4: Sweden	0.371
4	A_6: France	0.336
5	A_3: Germany	0.302
6	A_1: Italy	0.245

In the graphic representation of the total pseudo order there are not any differences in regard with the other preference structures and a constant feature is the Incomparabilities elimination, with the advantages and disadvantages that this implies.

7. Final conclusions

They can be obtained very important conclusions with regard to the functioning of the PROMETHEE Methods under the New Preference Structures.

The proposed methodology is particularly attractive since it allows working together wit: generalized criteria, outranking relations and new preference structures to solve Multicriteria Decision problems. Being considered as an extension of the PROMETHEE Methods it fulfills the essential requisites that govern them and it consequently has an ample acceptance for being easily understandable.

The New Preference Structures which they work with, namely: semiorder, interval order and pseudo order, allow to a more realistic modelization of the decision maker's preferences. In this way, the decision maker can express more freedom his preferences without having to submit to the strictness of the traditional schemes, giving an extensive acceptability to the Partial Comparability Axiom of Preferences.

However, it cannot be removed some commentary with regard to the subjectivity that entails the incorporation of global thresholds to the analysis. For themselves, those preference structures defining thresholds or tolerance gap, incorporate a certain subjectivity to the analysis, since not always is really understood the meaning that they have. Even more, at working with thresholds stemming from the aggregation of a series of them, the risk of losing objectivity tends to be large.

On the other hand, the fact of introducing preference structures that allow dealing with Weak Preference, this is to say, the hesitation between Indifference and Strict Preference and with Intransitivity, notably improves the modelization task.

The new methodology tends to emphasize the Indifference role in the rankings and it is in this direction towards which the most recent researches are oriented. In the same way, the appearance of intransitive relations, actually very accepted, is facilitated. Among the positive aspects of the non-transitivity it recovers importance the one that assumes as unique explication, the imperfect discrimination capacity of the human mind because of this only recognizes the inequalities when they have enough magnitude. In effect, the theory of semiorder, interval order and pseudo order generalizes the concept of weak order to allow an imperfect discrimination when the options are near and the decision maker does not have more refined measurement tools.

It is very important to highlight that the wealthy utilization of generalized criteria for each criterion, with the object of obtaining a measure of the intensity of decision maker's preferences, leads to the adoption of new structures not much utilized in practice, for the modelization of his preferences. Such preference structures are: semiorder, interval order and pseudo order.

Finally, a larger interaction as well as an active decision maker's participation is obtained during the whole decision process. The decision maker becomes an essential and irreplaceable figure and besides depending on his identification with the problem in question, so it results a more realistic modelization of his preferences.

The truly novel results appear in the partial rankings, both of positive and negative flows, demanding consequently new graphs for the relation representations. In this way is how the graphs in correspondence with semiorders and pseudo orders supply a lot of information.

Although, they are visibly more complex than in the preorder cases, due to the great quantity of situations they allow the intransitivity and the weak preference, too.

However, the proposed methodology is not free of weak points that in several decisions make its application questionable.

Therefore the disadvantages attribute to the methodology are:

1. If the objective consists of solving the multicriteria decision problem by obtaining complete rankings, the new methodology does not bring anything new. Then, the final ranking obtained is the same whichever is the preference structure under study.

2. Even though the decision-maker perfectly understands the new methodology, there is no doubt that the preference structures this methodology works with make it more complex reducing its attractiveness.
3. The detail application needs more time in spite of being computerized and requires a lot of additional information not always available with the simplicity and the precision that were desirable.

A whole analysis of the advantages and the disadvantages makes possible to conclude that the methodology will be extensively accepted when the decision-maker familiarizes himself with it and he directs the goodness of general character in his own interests, trying to minimize the cost of the disadvantages.

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