Revista Electrónica de Comunicaciones y Trabajos de ASEPUMA. **Rect@**
Volumen 20(2019). Páginas 153 a 166.
DOI: 10.24309/recta.2019.20.2.03

LIQUIDITY MEASURES FOR BONDS SELECTION TO ESTIMATE THE INTEREST RATE CURVE: SPANISH CASE

SONIA RODRÍGUEZ-SÁNCHEZ*
sonia.rodriguezsanchez@ceu.es
Applied Mathematics and Statistics Department, School of Economics,
CEU San Pablo University,
Julian Romea, 23. 28003 Madrid (Spain)

MARIANO GONZÁLEZ-SÁNCHEZ
mariano.gonzalez@cee.uned.es
Departamento Economa de la Empresa y Contabilidad, Facultad de Economa,
Universidad Espaola a Distancia (UNED)

M. CARMEN GARCÍA-CENTENO
garcen@ceu.es
Applied Mathematics and Statistics Department, School of Economics,
CEU San Pablo University,
Julian Romea, 23. 28003 Madrid (Spain)

Recibido (19/09/2019)
Revisado (22/10/2019)
Aceptado (28/10/2019)

RESUMEN: La Autoridad Europea de Seguros y Pensiones de Jubilación establece que la estimación de la curva de tipos de interés, utilizada para la valoración de las operaciones de las compañías de seguros, debe considerar todos los bonos líquidos; en particular, para la zona Euro, recomienda usar bonos con vencimiento hasta 20 años como último punto líquido. La literatura financiera ha analizado los diferentes componentes de la liquidez (rigidez, inmediatez, amplitud, resistencia y profundidad) y ha encontrado una relación significativa entre el binomio de riesgo y rendimiento con la liquidez. Este documento busca los indicadores de liquidez correlacionados con el rendimiento y el riesgo de los bonos, para seleccionar el último punto líquido. En una muestra diaria de datos del mercado de Deuda Pública Española, encontramos que, hasta un mes, los indicadores de profundidad y amplitud son significativos, mientras que a largo plazo, los indicadores significativos son amplitud y resilencia. Finalmente, de acuerdo con estos indicadores, la relación riesgo-liquidez conduce a un punto líquido final de 7 años, mientras que en la relación rendimiento-liquidez es de 5 años.

Palabras Clave: liquidez; rigidez; profundidad; amplitud; resilencia; último punto líquido

ABSTRACT: The European Insurance and Occupational Pensions Authority establishes that the estimation of the interest rate curve, used for the valuation of insurance company operations, should consider all liquid bonds; in particular, for the euro zone, if fixes bonds with maturity up to 20 years as the last liquid point. The financial literature has analyzed the different components of liquidity (rigidity, immediacy, breadth, resilience and depth) and it has found a significant relationship between the return-risk binomial and liquidity. This paper searches for the liquidity indicators correlated with bond yield and risk, in order to select the last liquid point. On a daily data sample of the Spanish public debt market we find that, up to one month, the indicators of depth

Keywords: liquidity; rigidity; depth; amplitude; resilience; last liquid point.

*doctoral candidate CEINDO-CEU, Economics and Law Program

1. Introduction and Background

Following Directive 2009/138/EC (hereinafter, Solvency-II), which includes the regulation of the insurance sector in the European Union (EU), it is necessary for insurance companies to apply an economic valuation of the entire balance, in which all assets and liabilities are valued according to market principles. Therefore, the term structure of risk-free interest rates becomes a key element for this valuation by insurance and reinsurance companies.

On the other hand, Regulation 1094/2010 of the European Parliament and the Council creates the European Insurance and Occupational Pensions Authority (hereinafter EIOPA). This EU agency has, among its technical functions, the establishment of technical criteria for the estimation of the interest rate curve. Within these requirements, we highlight, as an objective of this study, the use of the most liquid bonds for estimating zero coupon curve. Thus, regardless of the method of estimating the term structure of interest rates, the EIOPA determines that the selection of debt instruments involved in the curve estimation is subject to the so-called last liquid point, which depends on the criteria of the residual volume. In particular, this selection depends on the currency so, for example, for the Euro the last liquid point is set at 20 years.

In financial literature, most authors agree that liquidity is the second most important factor, after credit risk, that affects bond yields. Since the pioneering work [6], which showed liquidity as the variable responsible for the differences between the profitability between private equity securities and public securities, many authors have tried to study the debt liquidity from the market data.

Now, the financial literature has studied the liquidity of the markets under other parameters that expire, such as the continuity in the negotiation of securities, transaction costs or negotiated volume. So, [1] and [2] study corporative bonds liquidity risk; [3] and [4] analyze liquidity differences between main USA corporative bonds before and after recent financial crisis. But, an asset is more liquid when the costs of undoing positions immediately are low (or the difference between the purchase price and the sale price, the bid-ask spread, is lowest) and the higher the number of orders necessary to modify prices, that is, as [5] points out, the capacity of the market to accommodate large trading volumes without causing high price variations.

Some papers state that the first consequence of the liquidity factor is the higher demand for profitability on the part of investors, as [7]. These differences are known as liquidity premiums and they are deviations from the returns of the various assets to compensate for liquidity differences. This liquidity premium has been frequently discussed in the literature, for example, [7], [8], [9] and [10], with the objective of analyzing the relationship between yield and liquidity.

An additional issue raised in the literature is the way in which liquidity is measured. For public debt, some studies, as [9], indicate that liquidity should be closely linked to the inventory risk and the costs of processing the orders, which ultimately depend on the level of risk of the asset and the transactions execution frequency. Other works, for example [11], show that bond issues, which more recently were auctioned, tend to be more liquid than those of bonds issued previously, although the maturity dates are similar. However, even among these bonds there could be differences in liquidity. Therefore, liquidity is also related to the asset risk

In this line, and for short-term interest rates, some works, as [12], have considered the duration component of liquidity, in order to capture the relationship between liquidity and risk; although they do not refer to the volume of negotiation. There is also another, for example [13], similar approach on the heteroscedasticity of interest rates, but in the form of a time series instead of a cross-section of liquidity. With respect to trading volume as an explanatory variable of the liquidity differences of the bonds of the same issuer, as [8], and given the unavailability of this variable with a regular frequency, it is usual to replace it with the seniority of the bonds.

In summary, the literature shows that the liquidity of the bonds is related to, at least, two fundamental factors: yield and risk; hence the selection of liquidity indicators should take these interrelations into consideration.

In this context, the objective of this paper is to check whether the liquidity indicators developed by the financial literature for stock markets can be applied to the bond markets, in order to allow selecting the most liquid ones that can be subsequently used to estimate the zero coupon curve required by Solvency-II. But, liquidity indicators are traditionally designed for the stock market and then, their results are not conclusive for bond managers since their objectives and strategies sometimes differ from the agents that operate in the equity markets. Therefore, our study is fully justified to try to identify which indicators are more related to basic and habitual measures of profitability and risk in the bond markets. Additionally, our objective is to consider an isolated country to avoid sovereign risk different from each of them. We consider that estimating a curve for the Euro, without taking into account the different sovereign risk, it is a topic to be dealt with in another paper.

With this objective, the work is structured as follows: in section 2 establishes the empirical methodology followed to test the validity of these indicators in a bond market; in section 3 the sample is described; section 4 contains the results and section 5 are the main conclusions of the study.

2. Methodology

2.1. *Measures of market liquidity*

Measuring market liquidity is not an easy task, since its definition includes several dimensions. From [14], five dimensions or characteristics, as the standard for a liquid market, are studied: rigidity, immediacy, amplitude, resilience and depth.

The concept of rigidity refers to transaction costs, which are assumed to be low in liquid markets. However, one of the most common indicators to measure rigidity is the ask-bid spread, so that the lower the spread is, the better the liquidity conditions are. However, as [15] notes, the high volatility of this indicator in certain occasions, implies that rigidity is also estimated following other indicators such as [16]. This high-low spread estimator reflects both the true variance of the share price and the ask-bid spread. But, while the variance component grows proportionally with the term, the dispersion component does not, thus avoiding excessive dispersion of data. [16] is estimated as follows:

$$
\begin{aligned}
CS &= \left| \frac{2\exp\alpha - 1}{1 + \exp\alpha} \right| \\
\alpha &= \frac{\sqrt{2\beta} - \sqrt{\beta}}{3 - 2\sqrt{2}} - \sqrt{\frac{\gamma}{3 - 2\sqrt{2}}} \\
\beta &= \frac{1}{2} \sum_{j=0}^{h} \left(\ln \frac{Mx_{t+j}}{Mn_{t+j}} \right)^2 \\
\gamma &= \left(\ln \frac{Mx_{t,t+h}}{Mn_{t,t+h}} \right)^2
\end{aligned}
\tag{1}
$$

Where CS is the indicator value, Mx_t is maximum price of the day t, Mn_t is minimum price of the day t, $Mx_{t,t+h}$ is the highest price among maximum prices for daily period from t to $t + h$, $Mn_{t,t+h}$ is lowest price among minimum prices for daily period from t to $t + h$. So, the higher the value of the indicator, the greater the rigidity of the market.

On the other hand, immediacy (I) shows whether operations are executed quickly and orderly; it is usually measured by the daily price range, that is, the difference between the highest and lowest price of an asset during a trading day. When this difference is large, immediacy is low and operations become more difficult to implement or can lead to huge price movements once executed. Therefore, large oscillations of the daily range suggest a weak immediacy. Another indicator that can help us to measure immediacy is[17]. This indicator (I_R) is based on the serial covariance of the change in the price of assets, so that lower the relationship between the price changes of two consecutive dates, higher the immediacy:

$$I_R = 2\sqrt{\left|cov\left(\Delta P_t, \Delta P_{t-h}\right)\right|} \tag{2}$$

The amplitude measures the impact of orders on prices. Among the usual indicators to measure it, [18] stands out, which is the absolute return on the volume for the day t estimated as:

$$A = \frac{1}{h}\sum_{j=0}^{h-1}\frac{|r_{t-j}|}{v_{t-j}} \tag{3}$$

Where h is size of estimation period, v is daily volume and r is the first difference of log daily prices or return.

Another indicator of the amplitude is [19] defined as:

$$J = \sqrt{\sum_{k=1}^{N_t}\frac{v_{k,t}}{v_t}\left(\frac{P_{k,t} - \bar{P}_t}{\bar{P}_t}\right)^2} \tag{4}$$

Where N is the total number of daily operations on the asset, v is daily volume of day t, $P_{k,t}$ is the price of transaction k in day t, $v_{k,t}$ is the volume corresponding to transaction, and $\bar{P}_t$ is mean price of all transactions of day t. So, the higher this indicator, the higher the amplitude.

According [20], resilience measures whether prices can move quickly to new levels of equilibrium, and therefore, is closely related to market efficiency. Usually, from [21], the Market Efficiency Coefficient (MEC) is used to measure resilience. As [14] points out, this indicator is a relationship between the variance of long-term and short-term returns:

$$MEC = \frac{var\left(R_t\right)}{h \cdot var\left(r_t\right)} \tag{5}$$

Where var is the variance function, R and r are the long (h) and short term returns, respectively, estimated as the difference of the logarithms of the prices. Thus, if a market is resilient, short and long-term volatilities are similar, with prices moving faster to new equilibrium levels and, therefore, the MEC ratio should be close to one in the resilient markets and deviate from unity in markets characterized by poor resilience.

Finally, the depth is linked to the number of orders, and the measures used to measure this property of the markets are usually related to the trading volume or the turnover rate, defined as the volume of trading on the size of the market (measured for the living debt). Thus, a low turnover means that only a small part of this market is negotiated at each time, which would indicate a low level of market liquidity. But there are other indicators that refer to the abundance of orders in the market such as [22]; thus, this indicator (L) is estimated for a sample period of h days, such as the number of days without negotiation (c) over the total, this is $L = \frac{c}{h}$. This means that the indicator is closer to one, when the bond is less liquid.

2.2. *Selection of liquidity measures*

According to literature reviewed, empirical studies on liquidity have found a relationship between liquidity and return-risk binomial of financial assets. Therefore, our proposal for the selection of liquidity indicators goes through the liquidity indicators that are related to the yield and risk of the bonds. Thus, we use internal rate of return (IRR) as a measure of bond yields and, as a first-order approximation of risk, we use the bond duration or D defines for [23], which measures the sensitivity of the bond price of parallels movements of term structure of interest rate[1]

[1] We do not use Modified Duration since similar to [23] and include the other dependent variable. We do not use Fisher Weil duration since a priori needs a zero coupon curve and then, the results could be affected by model risk. Finally, we do not use Effective duration because is discrete expression applies usually for bond with embedded options.

Consequently, given that liquidity is a component of the bond price (P) and that the relationship between price variations (Δ) and their corresponding D and IRR is linear ($\Delta P = \frac{D}{1+IRR} \cdot \Delta IRR$) then, the general equation to select liquidity measures have the following matrix expression:

$$y_{i,t} = \alpha + X_{i,t}^T \cdot \beta + u_{i,t} \tag{6}$$

Where $i = 1, \cdots, N$ is each of the bonds traded each day ($t = 1, \cdots, T$) of the total sample analyzed, $y_{i,t}$ is the dependent variable of each bond i and on each date t, in our case they are two variables (IRR and D), $x_{i,t}^T$ is the vector of independent variables of the model or liquidity indicators of each bond in each moment. In this way, if for any indicator x_k, we obtain a β_k statistically significant, then this indicator is selected to measure the liquidity of the bond market.

Ec.(6) is a panel data model and, $u_{i,t}$ is the random perturbation that we are going to assume follows a Normal distribution with zero mean and constant variance equal to σ^2 (homocedasticity). It is assumed that there is no temporal or spatial correlation, that is, $cov(u_{i,t}, u_{k,t}) = 0$, $\forall i \neq j$ (non-autocorrelation) and that exogeneity is satisfied, i.e., $cov(u_{i,t}, x_{k,t}) = 0$, $\forall i, k, t$.

The estimation of the panel data model proposed, in addition to the above, has to consider several relevant issues. First, the standard errors of the parameters must be consistent with heteroscedasticity and autocorrelation, since these drawbacks are common when a large data set and long time series are available, see for example [24].

Another relevant issue is the possible endogeneity, to solve this problem we decompose the residuals of the model into δ_i a component relative to each individual i (in our case, relative to each bond) that remains stable over time and, $\epsilon_{i,t}$ a component related to variation between individuals and through temporary instants. This decomposition is known as fixed effects and for its estimation it is enough to include as many dummies as bonds and use Ordinary Least Squares (OLS). Now, as [25] and [26] point out, if the individual effects are not constant over time then the panel shows random effects, in which case the appropriate estimation method is Generalized Least Squares (GLS). To discriminate which of the two models we have to estimate, we apply the usual Hausman test from [27], whose null hypothesis is that the GLS estimates are consistent, so that under the null hypothesis the statistic follows the distribution χ^2 with degrees of freedom equal to the number of regressors.

Another disadvantage is the daily behavior of the dependent variables, both IRR and D are are non-stationary in mean, due to the tendency that they show. So, we define as dependent variable the first difference of the same, and to maintain the consistency of the model, we apply the same process with the dependent variables or liquidity indicators.

Finally, given that certain indicators depend on the sample period used for their estimation (Amihud, Lesmond, Jankowitsch, MEC and Roll), the panel data model is estimated for three different periods (5, 20 and 257 consecutive daily data or rolling estimation), in this way, we check the best liquidity indicators for each term, weekly, monthly and annually, respectively.

3. Data and Liquidity Measures

The data used in the analysis of the liquidity of sovereign bonds are Treasury Bills, Bonds and Liabilities issued by the Spanish State and outstanding at the end of 2017. The Strips have not been included due to their special characteristics. So, our sample consists of 55 income assets, the assets expired from January 2018 to July 2066. For each of which, we obtain their characteristics and daily data of Bloomberg, since January of 2016 until December 2017. The daily variables are closing price, ask and bid prices, highest and lowest prices, as well as data of negotiated effective volume and nominal volume negotiated by the Bank of Spain.

In order to be able to analyze the liquidity of the bonds according to their maturity and in order to be able to fix the last liquid point to which the EIOPA refers when estimating the interest rate curve, we have ordered these 55 fixed income assets object of study in function of its expiration date, denominating, in this way, $B1$ the bond that expires in a period of time closer and $B55$ to the one of higher maturity (see Table-1).

Table 1. Characteristics of sample bonds

Code	Type	Cupon%	Expiration date	Currency	Issue date	code ISIN	Nominal per bond	Cupons
LT1	T-Bill	0.00	19-Jan-18	EUR	20-Jan-17	ES0L01801197	1000	0
B2	T-Bonds	4.50	31-Jan-18	EUR	13-Nov-12	ES00000123Q7	1000	6
LT3	T-Bill	0.00	16-Feb-18	EUR	17-Feb-17	ES0L01802161	1000	0
LT4	T-Bill	0.00	9-Mar-18	EUR	10-Mar-17	ES0L01803094	1000	0
LT5	T-Bill	0.00	6-Apr-18	EUR	7-Apr-17	ES0L01804068	1000	0
B6	T-Bonds	0.25	30-Apr-18	EUR	26-May-15	ES00000127D6	1000	3
LT7	T-Bill	0.00	11-May-18	EUR	12-May-17	ES0L01805115	1000	0
LT8	T-Bill	0.00	15-Jun-18	EUR	16-Jun-17	ES0L01806154	1000	0
LT9	T-Bill	0.00	13-Jul-18	EUR	14-Jul-17	ES0L01807137	1000	0
B10	State Liabilities	4.10	30-Jul-18	EUR	19-Feb-08	ES00000121A5	1000	11
LT11	T-Bill	0.00	17-Aug-18	EUR	18-Aug-17	ES0L01808176	1000	0
LT12	T-Bill	0.00	14-Sep-18	EUR	15-Sep-17	ES0L01809141	1000	0
LT13	T-Bill	0.00	12-Oct-18	EUR	13-Oct-17	ES0L01810123	1000	0
B14	T-Bonds	3.75	31-Oct-18	EUR	9-Jul-13	ES00000124B7	1000	6
LT15	T-Bill	0.00	16-Nov-18	EUR	17-Nov-17	ES0L01811162	1000	0
LT16	T-Bill	0.00	7-Dec-18	EUR	8-Dec-17	ES0L01812079	1000	0
B17	T-Bonds	0.25	31-Jan-19	EUR	26-Jan-16	ES00000128A0	1000	3
B18	T-Bonds	2.75	30-Apr-19	EUR	14-Jan-14	ES00000124V5	1000	6
B19	State Liabilities	4.60	30-Jul-19	EUR	10-Feb-09	ES00000121L2	1000	11
B20	State Liabilities	4.30	31-Oct-19	EUR	2-Jun-09	ES00000121O6	1000	11
B21	T-Bonds	1.40	31-Jan-20	EUR	8-Jul-14	ES00000126C0	1000	6
B22	State Liabilities	4.00	30-Apr-20	EUR	20-Jan-10	ES00000122D7	1000	11
B23	T-Bonds	1.15	30-Jul-20	EUR	16-Jun-15	ES00000127H7	1000	6
B24	State Liabilities	4.85	31-Oct-20	EUR	13-Jul-10	ES00000122T3	1000	11
B25	T-Bonds	0.05	31-Jan-21	EUR	6-Jun-17	ES00000128X2	1000	4
B26	State Liabilities	5.50	30-Apr-21	EUR	24-Jan-11	ES00000123B9	1000	11
B27	T-Bonds	0.75	30-Jul-21	EUR	8-Mar-16	ES00000128B8	1000	6
B28	State Liabilities	5.85	31-Jan-22	EUR	22-Nov-11	ES00000123K0	1000	11
B29	T-Bonds	0.40	30-Apr-22	EUR	24-Jan-17	ES00000128O1	1000	6
B30	T-Bonds	0.45	31-Oct-22	EUR	10-Oct-17	ES0000012A97	1000	6
B31	State Liabilities	5.40	31-Jan-23	EUR	29-Jan-13	ES00000123U9	1000	10
B32	State Liabilities	4.40	31-Oct-23	EUR	21-May-13	ES00000123X3	1000	11
B33	State Liabilities	4.80	31-Jan-24	EUR	16-Sep-08	ES00000121G2	1000	16
B34	State Liabilities	3.80	30-Apr-24	EUR	29-Jan-14	ES00000124W3	1000	11
B35	State Liabilities	2.75	31-Oct-24	EUR	20-Jun-14	ES00000126B2	1000	11
B36	State Liabilities	1.60	30-Apr-25	EUR	27-Jan-15	ES00000126Z1	1000	11
B37	State Liabilities	4.65	30-Jul-25	EUR	24-Feb-10	ES00000122E5	1000	16
B38	State Liabilities	2.15	31-Oct-25	EUR	9-Jun-15	ES00000127G9	1000	11
B39	State Liabilities	1.95	30-Apr-26	EUR	19-Jan-16	ES00000127Z9	1000	11
B40	State Liabilities	5.90	30-Jul-26	EUR	15-Mar-11	ES00000123C7	1000	16
B41	State Liabilities	1.30	31-Oct-26	EUR	26-Jul-16	ES00000128H5	1000	11
B42	State Liabilities	1.50	30-Apr-27	EUR	31-Jan-17	ES00000128P8	1000	11
B43	State Liabilities	1.45	31-Oct-27	EUR	4-Jul-17	ES0000012A89	1000	11
B44	State Liabilities	5.15	31-Oct-28	EUR	16-Jul-13	ES00000124C5	1000	16
B45	State Liabilities	6.00	31-Jan-29	EUR	15-Jan-98	ES0000011868	1000	30
B46	State Liabilities	1.95	30-Jul-30	EUR	4-Mar-15	ES00000127A2	1000	16
B47	State Liabilities	5.75	30-Jul-32	EUR	23-Jan-01	ES0000012411	1000	30
B48	State Liabilities	2.35	30-Jul-33	EUR	1-Mar-17	ES00000128Q6	1000	17
B49	State Liabilities	4.20	31-Jan-37	EUR	17-Jan-05	ES0000012932	1000	32
B50	State Liabilities	4.90	30-Jul-40	EUR	20-Jun-07	ES00000120N0	1000	34
B51	State Liabilities	4.70	30-Jul-41	EUR	28-Sep-09	ES00000121S7	1000	32
B52	State Liabilities	5.15	31-Oct-44	EUR	16-Oct-13	ES00000124H4	1000	32
B53	State Liabilities	2.90	31-Oct-46	EUR	15-Mar-16	ES00000128C6	1000	31
B54	State Liabilities	4.00	31-Oct-64	EUR	8-Sep-14	ES00000126D8	1000	51
B55	State Liabilities	3.45	30-Jul-66	EUR	18-May-16	ES00000128E2	1000	51

Then, to measure the liquidity of these assets, all liquidity indicators have been calculated for each bond and on all days of 2016 and 2017[2]. Due to the large volume of data involved, we have made a descriptive analysis to draw conclusions about their liquidity from the average accumulated values of each indicator during the sampling period and for each bond (see Table-2).

First, we note that the bonds that present the best levels of liquidity are the Treasury Bills (short term), although in some cases, are also bonds with maturity over one year.

To analyze the rigidity, the ask-bid liquidity indicator shows that the bond with lowest liquidity is $B54$, which was issued in 2014 and expires in 2064, it is a bond that, in the period considered, shows many days whitout negotiation, which makes the value of its liquidity indicators very volatile. Besides, statistics show that the bonds that have higher values, and therefore less liquid, are the bonds whose maturity is higher. Particularly from $B43$, which expires in 2027, which could be considered as the last liquid point. In addition, these bonds have the lowest coefficient of variation (except $B54$), so we can consider their most representative average values.

As for immediacy, the spread between the maximum and the minimum prices, that is, the price range, shows very constant results for all terms. So, if we place the last liquid point from 25% of the less liquid bonds, again $B43$ is last liquid point, which expires in October 2027.

[2]We have considered only two years giving stability to the sample of bonds, since as we increase the sample size there are many expirations and new issues that would distort the results.

Table 2. Empirical quartiles of liquidity measures

Term	Quartile	Rigidity		Immediacy		Amplitude		Resilience	Depth
		Ask-bid	Corwin-Schultz	High-low	Roll	Amihud	Jankowistch	MEC	Lesmond
5 days	Min.	0.0003 (LT16)	-0.0001 (LT16)	0.0005 (B23)	0.0024 (LT4)	0.0001 (LT15)	0.0001 (LT7)	0.0114 (LT8)	0.001 (LT16)
	Q1	0.0337 (LT9)	-0.0005 (B30)	0.0407 (B28)	0.0199 (B17)	0.0057 (B1)	0.0007 (B14)	0.0084 (LT4)	0.1706 (B32)
	Q2	0.0751 (B24)	-0.0021 (B43)	0.2015 (B35)	0.0594 (B27)	0.1049 (B14)	0.0024 (B29)	0.0072 (B30)	0.2852 (B17)
	Q3	0.119 (B43)	-0.0052 (B48)	0.5646 (B43)	0.1627 (B39)	0.6259 (B26)	0.0062 (B39)	0.0052 (B32)	0.3632 (B34)
	Max.	1.5025 (B54)	-0.0138 (B55)	1.6178 (B55)	12.130 6 (B54)	6.1056 (B45)	0.8851 (B54)	0.0036 (B47)	0.6874 (B54)
20 days	Min.	0.0001 (LT16)	-0.0001 (LT16)	0.0002 (B30)	0.0076 (LT1)	0.0032 (LT16)	0.0004 (LT7)	0.0045 (B2)	0.001 (LT16)
	Q1	0.0332 (LT9)	-0.0005 (B30)	0.0407 (B36)	0.0913 (B14)	0.0082 (B19)	0.0043 (LT15)	0.0025 (B20)	0.1754 (B25)
	Q2	0.0749 (B24)	-0.0021 (B43)	0.2017 (B40)	0.1958 (B27)	0.1135 (B22)	0.0092 (B27)	0.0021 (B38)	0.2992 (B17)
	Q3	0.1193 (B43)	-0.0052 (B48)	0.5677 (B43)	0.5093 (B39)	0.634 (B26)	0.0243 (B39)	0.0015 (B49)	0.3867 (B47)
	Max.	1.5198 (B54)	-0.014 (B55)	1.6211 (B52)	19.916 (B54)	6.3476 (B45)	1.752 (B54)	0.0007 (B54)	0.9549 (B54)
257 days	Min.	0.0002 (LT16)	-0.0001 (LT16)	0.0003 (B20)	0.0986 (B6)	0.0057 (LT16)	0.0208 (B6)	0.0386 (LT10)	0.1453 (B19)
	Q1	0.0195 (LT9)	-0.0004 (B30)	0.0351 (B29)	1.1095 (B19)	0.0811 (B18)	0.17 (B19)	0.0246 (B19)	0.2819 (B28)
	Q2	0.0719 (B24)	-0.0022 (B43)	0.2168 (B37)	1.59 (B34)	0.5772 (B25)	0.2122 (B35)	0.0205 (B38)	0.4017 (B37)
	Q3	0.119 (B43)	-0.0054 (B48)	0.592 (B43)	3.1688 (B44)	1.5565 (B33)	0.3972 (B45)	0.0141 (B48)	0.457 (B42)
	Max.	1.6262 (B54)	-0.0173 (B55)	1.7242 (B52)	36.156 (B54)	6.3476 (B45)	2.1713 (B54)	0.0019 (B55)	0.9725 (B54)

An indicator of liquidity, which we can consider between the two previous measures, is the Corwin-Schultz estimator. According to this, if we look at the average and the quartiles, the last liquid point ($Q3$) is $B48$ with maturity in July 2033.

The rest of the indicators depend on the term considered, which allows us to study liquidity in three scenarios. Thus, for the weekly term, according to Amihud is $B26$ (April-2021) the last liquid point; for the Jankowitsch indicator the third quartile is $B39$ (April-2026), according to the Lesmond indicator it is $B34$ (April-2024), for Roll it is $B39$ (April-2026) and for MEC it is $B32$ (October-2023).

For monthly terms, the last liquid points are kept except for Lesmond, which becomes $B47$ (July-2032) and MEC, which is $B49$ (January-2037).

Finally, for one year sample period, the indicators change, with a clear increase in expiration as the sample period increases. In particular, the Amihud indicator shows that $B33$ (January-2024) is the third quartile; for Jankowitsch this critical point is situated at $B45$ (January-2029); according to Lesmond it is $B42$ (April-2027); for MEC it is $B48$ (July-2033); and, Roll places the point in $B44$ (April-2028).

In Table-3 we show the last liquid point, according to each indicator and term, which corresponds with bond on the third quartile[3]. As we can see, the minimum maturity is around 5 years, the average is about 10 years and the maximum next to 20 years.

Table 3. Last liquid point for each liquidity measures

Term	Rigidity		Immediacy		Amplitude		Resilience	Depth
	Ask-bid	Corwin-Schultz	High-low	Roll	Amihud	Jankowistch	Corwin-Schultz	Lesmond
Week	B43 (Oct-2027)	B48 (Jul-2033)	B48 (Jul-2033)	B39 (Apr-2026)	B26 (Apr-2021)	B39 (Apr-2026)	B32 (Oct-2023)	B34 (Apr-2024)
Month	B43 (Oct-2027)	B48 (Jul-2033)	B39 (Apr-2026)	B39 (Apr-2026)	B26 (Apr-2021)	B39 (Apr-2026)	B49 (Jan-2037)	B47 (Jul-2032)
Year	B43 (Oct-2027)	B48 (Jul-2033)	B39 (Apr-2026)	B44 (Apr-2028)	B33 (Jan-2024)	B45 (Jan-2029)	B48 (Jul-2033)	B42 (Apr-2027)

In summary, we appreciate that all indicators, despite measuring different characteristics of market liquidity, find the same bond less liquid ($B54$), on the contrary, it is not so evident where the last liquid point is located, so that our study is fully justified, since to determinate this point, we use the liquidity indicator related to IRR and D of the bonds.

[3]We use quartile 3 to identify outliers as in usual tests (see Box Plot, Tukey, among others).

4. Empirical Results

First, we use Hausman test to analyze if the estimate should made by fixed effects or random effects. So, if null hypothesis is rejected then, the estimate by random effects is not consistent, therefore we use fixed effects.

Table 4. Hausmann test

	Duration	
Term	Hausman	p-value
Week	72.4076	0.0000
Month	68.7797	0.0000
Year	148.0681	0.0000
	IIR	
Week	47.2569	0.0000
Month	43.6616	0.0000
Year	26.563	0.0143

Table-4 shows, in all cases, the null hypothesis is rejected, since the p-value is less than 5% level of significance, therefore, the most appropriate estimation model is the panel data with fixed effects.

Next, for the duration, we estimate an unbalanced panel data model with fixed effects (for the different length of observations for each of the individuals), using as instruments a dummy variable for each day of the week, which avoids problems of autocorrelation (see Table-5).

Table 5. Results of panel data for Risk (D)

	Week			Month			Year		
	Coefficient	Std.Error	t-prob	Coefficient	Std.Error	t-prob	Coefficient	Std.Error	t-prob
Δ Bid-Ask	-0.0325	0.0292	0.2650	0.0575	0.2451	0.8150	0.0054	0.0178	0.7630
Δ Max-Min	-0.0015	0.0011	0.1690	-0.0274	0.0151	0.0690	-0.0494	0.0585	0.3980
Δ Corwin-S.	0.0466	0.0843	0.5800	-0.0608	0.1861	0.7440	0.0008	0.0012	0.4830
Δ Amihud	0.0740	0.4415	0.1675	0.0000	0.0001	0.6880	0.0001	0.0003	0.4930
Δ Lesmond	0.0125	0.0055	0.0240 (**)	1.8755	0.6642	0.0049 (**)	0.0002	0.0003	0.5290
Δ Jankowitsch	-0.3751	0.3095	0.2260	-0.0802	0.0340	0.0180 (**)	0.0019	0.0003	0.0000 (***)
Δ MEC	0.0144	0.0109	0.1870	0.1658	0.0521	0.0010 (***)	0.0001	0.0001	0.4820
Δ Roll	0.0029	0.0054	0.5920	0.2274	0.5816	0.6960	0.0014	0.0003	0.0000 (***)

Note: (*) = 10% of significance level; (**) = 5% of significance level; (***) = 1% of significance level.

For weekly period, where 10,632 observations have been used for the estimate, the only significant liquidity indicator is the Lesmond indicator. For the monthly term, with 10,189 observations, Lesmond, Jankowitsch and MEC are significant, and finally for the annual term, with 4,467 observations, they are Jankowitsch and Roll.

None of these models suffers from first or second order autocorrelation (see Table-6).

Table 6. Autocorrelation test for panel data of D

Test	Week	Month	Year
AR(1) test N(0,1)	1.0813 [0.860]	-1.2900 [0.197]	-0.5780 [0.282]
AR(2) test N(0,1)	0.3955 [0.692]	-1.6980 [0.090]	-0,2862 [0.775]

Note: between brackets is the p-value.

Next, we show the result of the panel data models with the variation of the IRR as a dependent variable in Table-(7),

For weekly term, with 10,632 observations, none of the indicators is significant, so we can affirm that they do not affect the variation of the IRR or, put another way, the yield of the bonds is not altered by the liquidity of the markets in weekly investments. On the other hand, both monthly (with 10,189 observations) and annual term (with 4,467 observations), the variation of the Market Efficiency Coefficient (ΔMEC) is statistically significant, for a level of significance of 1%. Therefore, the resilience of the market positively affects the yield of the bonds. Also, the Roll indicator is significant, although only 10%, for the annual frequency.

In addition, as it happened in the models of the variation of the duration, these panels do not suffer from autocorrelation of first or second order as can be seen in Table-8.

Table 7. Results of panel data for Yield (IIR)

	Week			Month			Year		
	Coefficient	Std.Error	t-prob	Coefficient	Std.Error	t-prob	Coefficient	Std.Error	t-prob
Δ Bid-Ask	0.0001	0.0002	0.3950	0.0001	0.0002	0.4570	-0.0001	0.0002	0.7010
Δ Max-Min	0.0001	0.0002	0.1880	0.0001	0.0002	0.1650	0.0001	0.0002	0.3650
Δ Corwin-S.	0.0006	0.0007	0.3940	0.0004	0.0007	0.5620	-0.0012	0.0010	0.2460
Δ Amihud	-0.0717	0.1096	0.5130	0.0112	0.3441	0.9740	-3.3801	3.1350	0.2810
Δ Lesmond	0.0002	0.0002	0.4660	-0.0001	0.0006	0.8680	0.0028	0.0154	0.8550
Δ Jankowitsch	0.0006	0.0029	0.8350	-0.0001	0.0019	0.9600	-0.0010	0.0010	0.2970
Δ MEC	0.0001	0.0002	0.4440	0.0029	0.0009	0.0020 (***)	0.2434	0.0550	0.0000 (***)
Δ Roll	0.0001	0.0002	0.2950	0.0002	0.0001	0.0530 (*)	0.0001	0.0003	0.9490

Note: (*) = 10% of significance level; (**) = 5% of significance level; (***) = 1% of significance level.

Table 8. Autocorrelation test for panel data of IIR

Test	Week	Month	Year
AR(1) test N(0,1)	0.82290 [0.411]	0.84380 [0.399]	0.74570 [0.456]
AR(2) test N(0,1)	0.9667 [0.334]	0.957 [0.399]	0.9639 [0.335]

Note: within brackets is the p-value.

From these results, and applying the same selection criterion, the third quartile of the signifcant liquidty indicator, for the weekly term, we find that the last point for the duration is $B34$, which expires in April 2024; in the monthly term, according to Lesmond, this point is $B47$ (January of 2032), according to Jankowitsch is $B39$ (April of 2026) and according to MEC is $B49$, which expires in January of 2037. This last issue is the only one consistent with the 20 years fixed by the EIOPA. However, in the annual term, based on the Jankowitsch indicator, the last liquid point is $B45$, which expires in January 2029 and, according to Roll is $B44$ (October 2028). As regards the IIR, the last liquid point, for monthly sample, according to the MEC, is given by the $B49$ bond, which expires in January 2037, also coinciding with the 20 years fixed by the EIOPA, but according to Roll is $B39$, which expires in April 2026 (less than 10 years from the end of our sample). If we study at the annual term, the last liquid point is $B48$, which expires in July 2033.

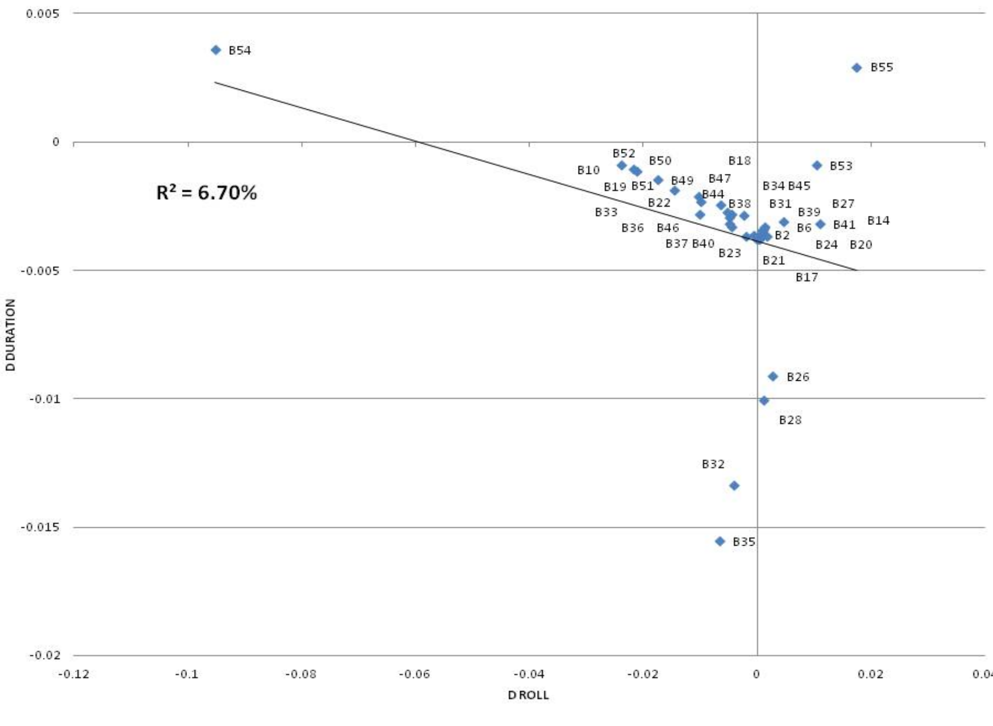

Figure 1. Regression of Duration variations on the Roll indicator variations for annual term and complete sample of bonds

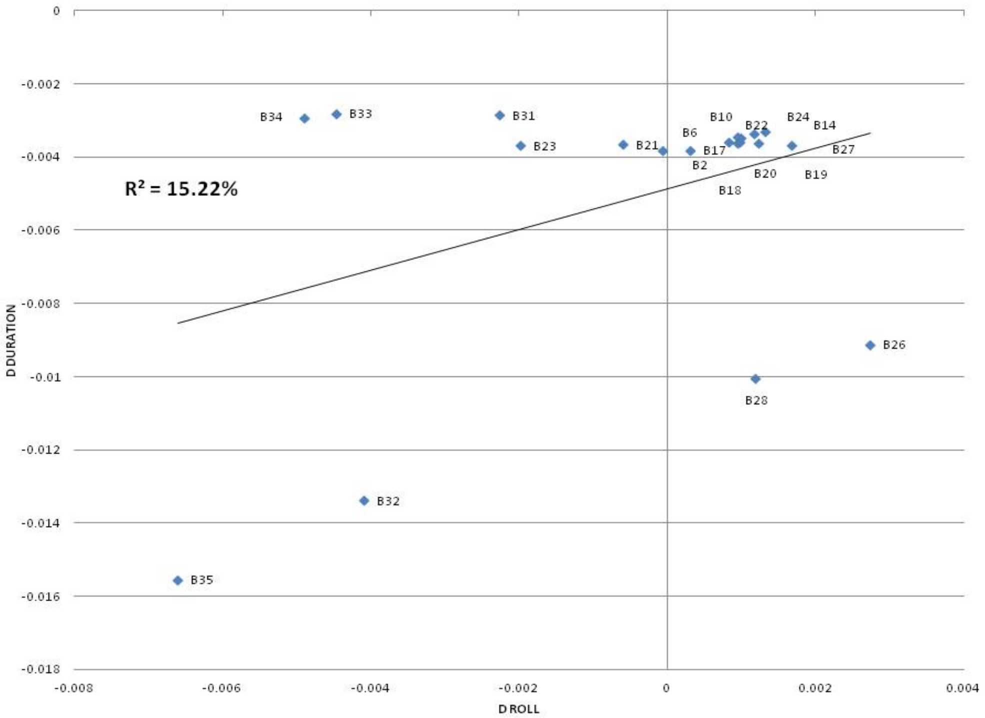

Figure 2. Regression of Duration variations on the Roll indicator variations for annual term and sample to last liquid point

So we study the direct relationship, by means of a linear regression on average values of the selected indicators and the variations of the IRR or D. In such a way that we select as the last liquid point the bond that, when excluding every bond after it from the regression, the statistical significance of the model, measured by the coefficient of determination (R^2), increases substantially.

For weekly period, the regression of the duration variations on the variation of Lesmond, shows an R^2 only of 0.13% for the total of bonds. However, if we consider the bonds from $B1$ to $B25$, the value of R^2 has a maximum of 3.30%, although it is still small, the difference with the total is significant (an increase of more than 2,000%) , so we consider the $B25$ bond, which expires in January 2021, as the last liquid point.

If we analyze the relationship between the variation of the duration and the variation of the liquidity indicators that have been significant, in the monthly and annual periods, the results are similar to obtained for weekly term. In particular, for monthly term, the regression of the variation of the duration on the variation of Lesmond, R^2 is 0.09%, considering the total of the sample, the value rises up to 10.09%, if we consider only up to the $B25$ bond. If the regression is on the variation of Jankowitsch, R^2 is 1.9% for the entire sample, and 10.65% with the bonds until the $B22$, which expires in April 2020. However, if we regress on the variation of the MEC for all the issues, R^2 is 0.04%, and 1.623% for the bonds up to $B35$ (it expires in January 2024).

For annual period, the regression of the variation of the duration on the variation of Jankowitsch, has a R^2 of 3.99%, with the total of the bonds, and 21.83% with the bonds up to the $B35$ (which expires January 2024), and up to 71.40% with the bonds up to $B18$ (April 2019). Similar results obtained when we regress on the variation of Roll, where R^2 goes from 6.70% to 15.22% if we consider only the bonds up to $B35$ (January 2024). Figure-1 and Figure-2 shows the results for Roll measure for year period.

Therefore, for the duration we consider that the last liquid point is determined by the $B35$ bond that expires in the year 2024, seven years after the end of our sample.

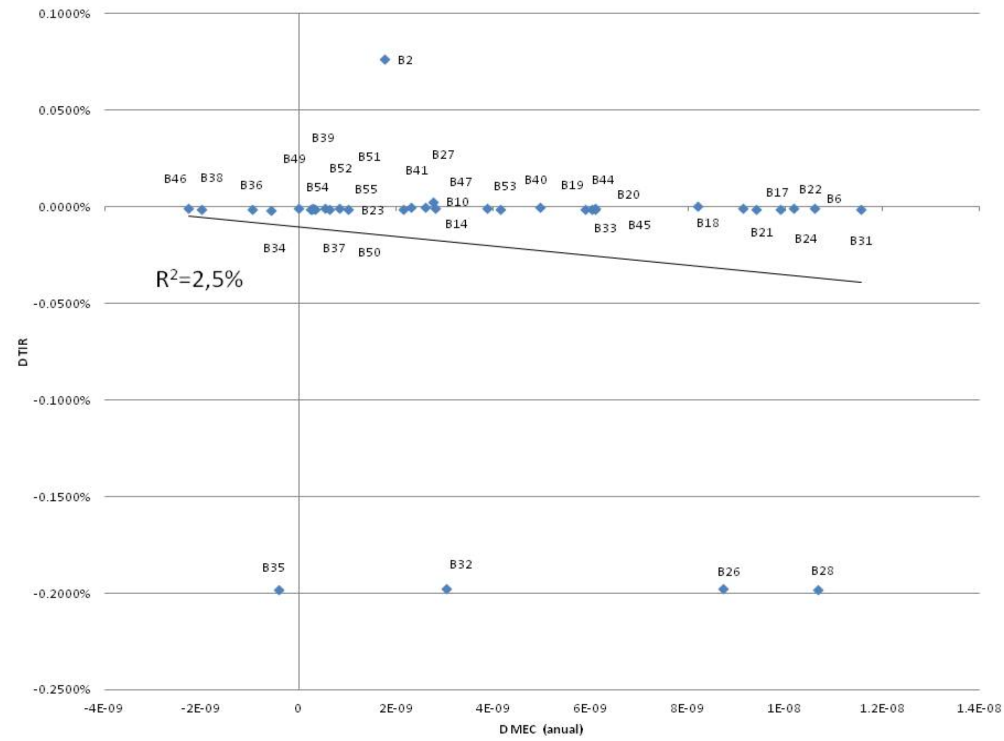

Figure 3. Regression of IIR variations on the MEC indicator variations for annual term and complete sample of bonds

If we analyze the direct relationship between the variation of IRR and the variation of the MEC, as the main explanatory indicator, for both monthly and yearly, we can see that in the monthly term and the total sample R^2 is 0.175%, which improves to 2.896% if we only consider the bonds until the $B28$ that expires in January 2022. The same happens in the regression on the variation of the Roll indicator, where R^2 goes from 0.955% to 19.4%.

For the annual term, the relationship between the variation of the IRR on the variation of the MEC, shows an R^2 equal to 2.5% when we take into account the total of the sample, this value improves up to 18.4% if we consider only the bonds up to $B28$ (see Figure-3 and Figure-4). Therefore, this is the bond that really marks us the last liquid point for IRR, with expiration in January 2022, or 5 years after the end of our sample, very far from the 20 years proposed by EIOPA.

5. Conclusions

Liquidity is fundamental when it comes to choosing bonds to estimate the interest rate curve and, more specifically, to determine the last liquid point (EIOPA).

The liquidity measures, traditionally used in stock markets, refer to five basic characteristics of liquidity, such as rigidity, immediacy, amplitude, depth and resilience. The usually indicators for these fundamentals measures of liquidity, we have focused on the ask-bid spread, the Corwin-Schultz indicator to measure the rigidity, the differential between the maximum and minimum price and the Roll indicator for measure the immediacy, the Amihud and Jankowitsch indicators to measure the amplitude, the Market Efficiency Coefficient to analyze the resilience and, finally, the Lesmond coefficient to measure the depth.

After the descriptive analysis of these liquidity indicators, applied to Spanish Public Debt negotiated at the end of 2017, we note that Treasury Bills (short term) are most liquid bonds, although in some cases, there is also a bonds of over one year. In addition, the bonds that have

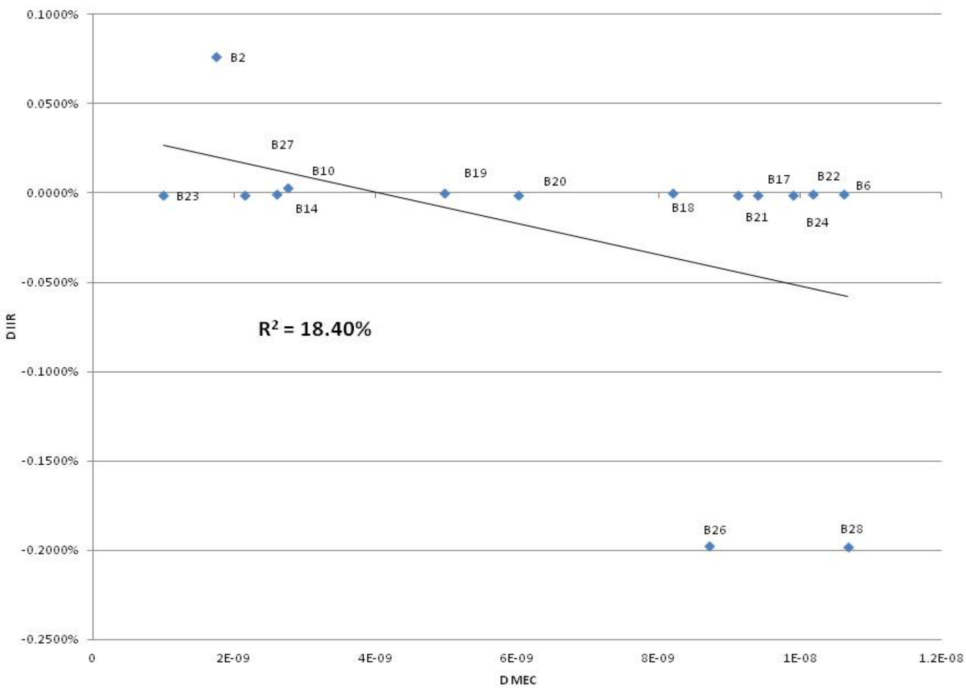

Figure 4. Regression of IIR variations on the MEC indicator variations for annual term and sample to last liquid point

higher values, and therefore less liquid, are the bonds whose maturity is higher, coinciding most of the indicators in which the least liquid bond is $B54$, which was issued in 2014 and expires in 2064, is a bond that, in the period considered, has many days in which it is not negotiated, which makes the value of its liquidity indicators very volatile.

On the other hand, the empirical work on liquidity has found a relationship between the return-risk binomial of financial assets and its liquidity. Thus, the election of bonds, for the determination of the last liquid point, must take into account those liquidity indicators that are related to the yield and risk of the bonds. In this sense, its internal rate of return (IRR) and its duration have been used in this work as a measure of bond yield and risk, respectively. In addition, this relationship is analyzed for three periods: weekly, monthly and annual.

In the weekly term, we have only found a direct relationship between the duration and depth of the market, measured by the Lesmond indicator. On the other hand, regarding yield (IRR) none of the liquidity indicators shows a significant statistical relationship.

For the period of one month, the liquidity indicators that affect the duration of the bonds are the Lesmond indicator (depth), the Jankowitsch indicator (amplitude) and the MEC (resilience). In cases of yield (IRR), MEC (resilience) and Roll (immediacy) indicators shows a positive relationship.

For the annual term, the indicators that have been statistically significant with respect to risk are Jankowitsch (amplitude) and Roll (immediacy). On the other hand, regarding yield, only MEC (resilience) presents a significant relationship.

In this way, and taking into account only the liquidity indicators that we have found significant to explain the interest rate risk (duration) and yield (IIR), we have selected the last liquid point in the Spanish sovereign bonds market as the bond with the longest maturity whose inclusion in the sample maximizes the coefficient of determination of the relation liquidity vs. yield and risk. Thus, the last liquid point is, for the risk, the bond with maturity close to 7 years, and for IRR, it is a bond with maturity close to 5 years. In short, from an empirical and statistical perspective, the election of the EIOPA of 20 years of maturity as the last liquid point of a generalized form for all issues in Euros does not seem justified.

Acknowledgement

This work has been supported by the Spanish Ministry of Economics and Competitiveness under grant MINECO/FEDER ECO2015-65826-P, and Cátedra Universidad CEU San Pablo-Mutua Madrileña insurance company (grant ARMEG 060516-USPMM-01/17).

References

1. V. Acharya, Y. Amihud, and S. Bharath, "Liquidity risk of corporate bond returns: Conditional approach", *Journal of Financial Economics* **110** (2013) 358–386.
2. Y. Amihud, A. Hameed, W. Kang and H. Zhang, "The illiquidity premium: International evidence", *Journal of Financial Economics* **117** (2015) 350–368.
3. J. Dick-Nielsen, P. Feldhutter and D. Lando, "Corporate bond liquidity before and after the onset of the subprime crisis", *Journal of Banking & Finance* **103** (2012) 471–492.
4. N. Friewald, R. Jankowitsch and M. Subrahmanyam, "Illiquidity or credit deterioration: A study of liquidity in the US corporate bond market during financial crises", *Journal of Financial Economics* **105** (1959) 18–36.
5. R. Pascual, "Liquidez: una revisión de la investigación en microestructura", *Revista de Economía Financiera* **1** (2003) 80–126.
6. L. Fisher, "Determinants of risk premiums on corporate bonds", *Journal of Political Economy* **67** (1959) 217–237.
7. Y. Amihud and H. Mendelson, "Asset pricing and the bid-ask spread", *Journal of Financial Economics* **17** (1986) 223–249.
8. E. Elton and T. Green, "Tax and liquidity effects in pricing government bonds", *The Journal of Finance* **53** (1998) 1533–1562.
9. F. Alonso, R. Blanco, A. del Río and A. Sanchís, "Estimating liquidity premia in the Spanish Government", *The European Journal of Finance* **10** (2013) 453–474.
10. A. Díaz, J. Merrick and E. Navarro, "Spanish Treasury bond market liquidity and volatility pre- and post-European Monetary Union", *Journal of Banking & Finance* **30** (2006) 1309–1332.
11. P. Pasquariello and C. Vega, "The on-the-run liquidity phenomenon", *Journal of Financial Economics* **92** (2009) 1–24.
12. Y. Amihud and H. Mendelson, "Liquidity, maturity, and the yields on us treasury securities", *The Journal of Finance* **46** (1991) 1411–1425.
13. A. Díaz, F. Jareño and E. Navarro, "Term structure of volatilities and yield curve estimation methodology", *Quantitative Finance* **11** (2011) 573–586.
14. A. Sarr and T. Lybek, "Measuring liquidity in financial markets", Working paper 02/232, Internacional Monetary Fund, Whasington-USA, 2002.
15. C. Brotos and M. Lamas, "Measuring market liquidity in US fixed income markets: A new synthetic", Documento de trabajo 1608, Bank of Spain, Madrid-Spain, 2016.
16. S. A. Corwin and P. Schultz, "A simple way to estimate bid and ask spreads from daily high and low prices", *The Journal of Finance* **67** (2012) 719–759.
17. R. Roll, "A simple implicit measure of the effective bid-ask spread in an efficient market", *The Journal of Finance* **39** (1984) 1127–1139.
18. Y. Amihud, "Illiquidity and stock returns: cross-section and time-series effects", *Journal of Financial Markets* **5** (2002) 31–56.
19. R. Jankowitsch, A. Nashikkar and M. Subrahmanyam, "Price dispersion in OTC markets: A new measure of liquidity", *Journal of Banking & Finance* **35** (2011) 343–357.
20. P. Bernstein, "Liquidity, stock markets, and market makers", *Financial Management* **2** (1987) 54–62.
21. J. Hasbrouck and R. Schwartz, "Liquidity and execution costs in equity markets", *The Journal of Portfolio Management* **14** (1988) 10–16.
22. D. Lesmond, J. Ogden and C. Trzcinka, "A new estimate of transaction costs", *Review of Financial Studies* **12** (1999) 1113–11141.
23. F. Macaulay, *Some theoretical problems suggested by the movement of interest rate, bond yield and stock price in U.S. since 1856* (Columbia University Press, New York-USA, 1938).
24. J. Woolridge, *Econometric Analysis of Cross Section and Panel Data* (2nd edition, MIT Press, Cambridge-USA, 1990).
25. W. Green, *Econometric Analysis* (3rd edition, Prentice Hall, New Jersey-USA, 1998).

26. B. Baltagi, *En Econometric Analysis of Panel Data* (3rd edition, Wiley, Chichester-UK, 2005).
27. J. Hausman, "Specification tests in econometrics", *Econometrica* **46** (1978) 1251–1271.