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IDENTIFYING ECONOMIC CYCLES IN SPAIN USING WAVELET FUNCTIONS: OIL PRICE, INDUSTRIAL PRODUCTION AND CONSUMER PRICE INDICES

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RESUMEN: Este artículo analiza la realidad cíclica de la macroeconomía española en base a tres variables relevantes y a lo largo del periodo temporal más amplio que nos ha permitido la disponibilidad de datos: Precio del Petróleo (1946M1-2015M9), Índice de Producción Industrial (1993M2-2015M9) e Índice de Precios al Consumidor (1961M1-2015M9). El impacto que ejerce el precio del petróleo en la economía ha sido estudiado extensa e intensamente, aunque modelizar sus efectos no es una cuestión trivial. Nuestra contribución se centra en la aplicación de funciones wavelet tipo Morlet para identificar la presencia de ciclos inestables en base a los datos conocidos mediante la computación de la Potencia Espectral Wavelet usando MATLAB. Adicionalmente, algunas técnicas bivariantes son útiles para visualizar la relación entre las tres variables consideradas. En concreto, la Coherencia Wavelet Cruzada a través de las diferentes fases puede usarse para detectar sincronismos y posibles relaciones de causalidad según bandas de frecuencia a lo largo del tiempo. Finalmente, los resultados obtenidos por otros autores para las economías de Estados Unidos y Alemania, en base a estas mismas variables y mismas técnicas con funciones wavelets, nos permiten añadir algunas conclusiones comparativas.

Palabras Clave: Ciclos Económicos, Precio del Petróleo, Economía Española, Potencia Espectral Wavelet, Coherencia Wavelet Cruzada.

ABSTRACT: This paper analyses the economic cycles in Spain over a long period of time according to available data by using three related variables: Oil Price (1946M1-2015M9), Industrial Production Index (1993M2-2015M9) and Consumer Price Index (1961M1-2015M9). The impact of shocks on oil price has been the subject of an extensive study, although modelling their effects is not a trivial undertaking. Our contribution focuses on applying the Morlet Wavelets to identify the presence of unstable cycles in data series by calculating the Wavelet Power Spectrum with the MATLAB software. Moreover, some bivariate techniques are applied to display the mutual influence of the Oil Price with the Industrial Production Index and the Consumer Price Index. The Cross Wavelet Coherency and the relationship among phases can also be used to detect co-movements and potential causality relationships in frequency bands over time. Finally, by studying these variables we can draw certain comparative conclusions with the US and German economies, whose corresponding variables have been considered by other authors using this same tool.

Keywords: Economic Cycles; Oil Price, Spanish Economy, Wavelet Power Spectrum, Cross Wavelet Coherency.

1. Introduction

Understanding the dynamics of economic cycles has been a main purpose for modern applied macroeconomics. The references on this topic have had among their objectives to define empirical regularities as a base for theoretical models and discriminate between alternative models. The study of economic cycles has had a major resurgence from the mid-1970s onward from the economic crises that developed countries experienced.

Wavelet analysis provides several tools with a high potential interest for understanding the relationships between economic variables involving heterogeneous agents making decisions over different time periods and operating at any given moment on different frequencies. A very interesting overview of the theory and practical applications of wavelet transform methods is presented in [1].

Our contribution is to identify economic cycles in Spain using a spectral approach, in particular, based on Morlet wavelets. This technique has been applied in [2, 3], among others, to study similar variables in the US economy up to 2007 and the German economy up to 2009, respectively. Also, for instance, [4] has used wavelet analysis to evidence the relationship between stock returns and aggregate economic activity in the US economy up to 2008. Later, [5] studies certain cycles in the Spanish macroeconomic, including six variables up to 2015, such as Oil Price, the Consumer Price Index (CPI) and others, but not including the Industrial Production Index (IPI), whose consideration was left as an open question. This study shows significant changes around 2008.

We now address the relationship between OIL Price, IPI and CPI in order to compare certain changes that have occurred in recent years in the Spanish economy with other economies.

We use the Discrete Wavelet Transform with two aims: first, to calculate the Wavelet Power Spectrum (WPS) of each variable; and second, to study the relationship between variables by calculating the Cross Wavelet Coherency (CWC).

Section 2 briefly discusses Wavelet versus Fourier analysis in order to justify the use of wavelet analysis for our purposes. Data sample is described in Section 3. Section 4 presents and comments on the results obtained from the WPS and density. Section 5 deals with the CWC. The computations were done with the MATLAB software [6], more specifically, in ASTOOLBOX included in [7]. Finally, we include the main conclusions and comparisons between the US, German and Spanish economies based on the three variables considered. Some main references have been included.

2. Wavelet versus Fourier analysis

Economic time series are an aggregation of components operating on different frequencies. Thus the question is how to decompose and understand this behaviour in any time series. Different tools exist and for the purposes of this paper we will use Wavelet instead Fourier analysis. The reason is that the former uses more flexible functions and it reveals how the different period components change over the time.

On the one hand, Fourier analysis, based on the use of cosine and sine functions, and the real and imaginary parts of certain functions such as $\psi(t) = e^{2ti}$ (Figure 1), can be used to study the cyclical nature of a time series in the frequency domain. However, under the Fourier transform, the time information of a time series is lost and it is hard to detect transient relationships or identify structural changes; moreover, it is only appropriate for stationary time series.

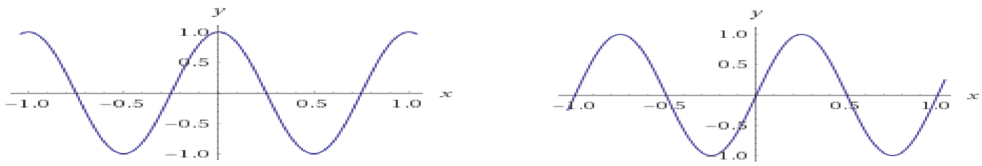

Figure 1. Real and imaginary parts of e^{2ti}.

If we have a signal x, identified by a time series x_t with N equally-spaced observations, the Discrete Fourier Transform (DFT) is of the form

$$X_m = \sum_{n=0}^{N-1} x_n e^{-i2\pi mn/N} \tag{1}$$

At the end of the process, the reconstructed signal will be

$$x_n = \frac{1}{N}\sum_{n=0}^{N-1} X_m e^{-i2\pi mn/N} \tag{2}$$

The concepts of period and frequency are relevant in this context.

On the other hand, wavelet analysis estimates the spectral characteristics of a time series as a function of time. It reveals how the different periodic components change over time. The wavelet function stretches into a long function to measure low-frequency movements. A wavelet function also compresses into a short function to measure high-frequency movements.

One of the most used wavelets in practice is the Morlet wavelet, which is of the form:

$$\psi(t) = \frac{1}{\sqrt[4]{\pi}} e^{6ti-t^2/2} \tag{3}$$

whose basic real and imaginary parts are shown in Figure 2.

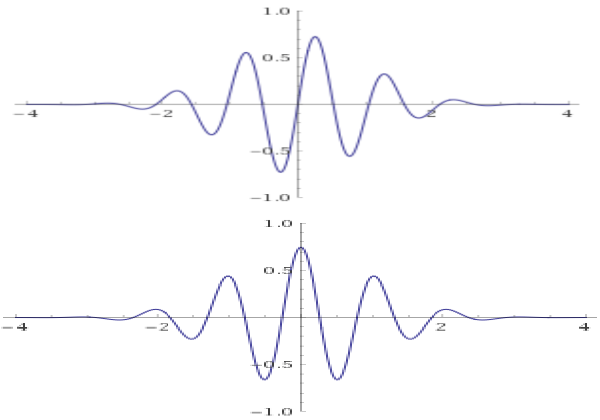

Figure 2. Real and imaginary parts of $\boldsymbol{e^{6ti-t^2/2}}$.

The DWT for the signal x follows this form:

$$W_m^x(s) = \frac{\delta_t}{\sqrt{s}} \sum_{n=0}^{N-1} x_n \psi\left((n-m)\frac{\delta_t}{s}\right), m = 0, \dots, N-1 \tag{4}$$

where δ_t is the Delta Dirac Function, and s is an important parameter called the Scaling Factor, which controls the changes in the signal’s frequency over time. This parameter does not exist in DFT. Now, we must consider three important concepts, namely frequency, period and scale. A directly proportional relationship exists between scale and period, as does an inverse relationship between scale and frequency.

The DWT is the basic tool that we use in this research. The wavelet transform decomposes a time series into sub-sequences at different resolution scales. It decomposes given data into high and low-frequency components. The wavelets can capture discontinuities, ruptures and singularities in the original data at high frequency (shorter time intervals). Also at low frequency (longer time intervals), the wavelet characterizes the data structure to identify long-term trends. Thus, the wavelet analysis allows us to extract the hidden and significant temporal features of the original data.

If we decompose a signal with DFT, in the final reconstructed signal the properties of the original signal are lost. Non-true cycles appear. However, this behaviour does not occur with DWT. This situation is shown schematically in Figure 3. In addition, DWT is computationally faster than DFT, even if DFT is calculated using Fast Fourier Transform (FFT).

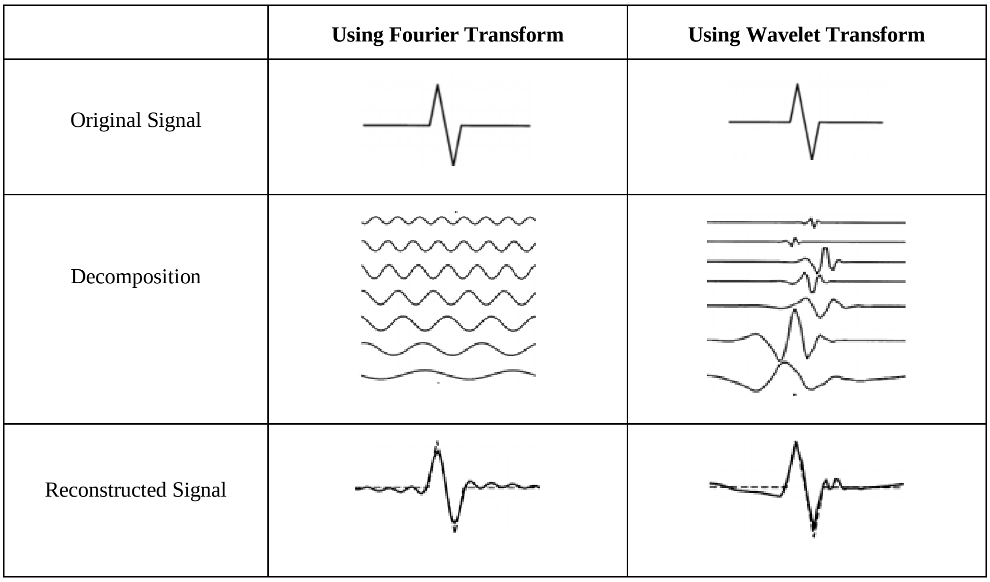

Figure 3. Schematic representation of a signal's decomposition and reconstruction using Fourier versus Wavelet transforms.

3. Data Description

Monthly data were gathered for economic variables that we refer to below. In fact, there are other types of variables that are also interesting in the analysis of economic cycles, such as tax collection, GDP, public expenditure or energy consumption. However, we have focused on these three relevant variables in order to provide comparative results with the ones given by [2, 3, 4] for United States and Germany, which are based indeed on the same variables and also use the techniques which have to do with wavelet functions.

(I) Crude Oil Price (OIL) (in Dollars per Barrel): Data are referred to the spot price of a barrel of benchmark crude oil, a reference price given by West Texas Intermediate (WTI) for buyers and sellers of crude oil. Prices are calculated by the U.S. Energy Information Administration (EIA) by taking an unweighted average of the daily closing spot prices. We take monthly growth rates considering the time period 1946M1-2015M9 and data provided by the Federal Reserve Bank of St. Louis.[1] These data are also accessed from the Independent Statistics & Analysis (EIA)[2]. In particular, sample data from the beginning of 1946 until the end of 2007 were analysed in [2].

According to the survey [8], most major oil price fluctuations dating back to 1973 can be explained by shifts in the demand for crude oil. As the global economy expands, so does demand for crude oil. The authors note that the price of oil has also increased at times due to greater demand for stocks (or inventories) of crude oil to guard against future shortages in the oil market. Both domestic political instability in oil producing countries and conflicts with other countries can destabilise the oil price (Korean War (1951–1953), Vietnam War (1950s – 1970s), Iranian Revolution (1979), 1990 Persian Gulf crisis and war, invasion of Iraq (2003), etc.)

The 2008 "Multiple Energy Market Initiatives" report from the U.S. Commodity Futures Trading Commission (CFTC) found that speculation had not caused significant changes in oil prices and that fundamental supply and demand factors provide the best explanation for the crude oil price increases. The report found that the primary reason for the price increases was that the world economy had expanded at

[1] http://research.stlouisfed.org/fred2/series/MCOILWTICO (U.S. Energy Information Administration (EIA), Crude Oil Prices: West Texas Intermediate (WTI) -Cushing, Oklahoma [MCOILWTICO], retrieved from FRED, Federal Reserve Bank of St. Louis)

[2] https://www.eia.gov/dnav/pet/hist/LeafHandler.ashx?n=PET&s=RWTC&f=M (Independent Statistics & Analysis, U.S. Energy Information Administration, EIA)

its fastest pace in decades, resulting in substantial increases in the demand for oil, while the oil production grew sluggishly, compounded by production shortfalls in oil-exporting countries. Consequently, as a result of the imbalance and low price elasticity, very large price increases occurred as the market attempted to balance a scarce supply against a growing demand, particularly in the last three years.

During 2014–2015, OPEC members consistently exceeded their production ceiling, and China experienced a marked slowdown in economic growth. At the same time, U.S. oil production nearly doubled from 2008 levels due to the fracking revolution (a technique of extraction by hydraulic fracturing). A combination of both factors led a plunge in U.S. oil import requirements and a record high volume of worldwide oil inventories in storage, and therefore a collapse in oil prices that continues into 2016.

(II) Industrial Production Index (IPI) (General Index, Base 2010): measures the monthly evolution of the Spanish productive activity of the industrial branches, that is, of the extractive, manufacturing, production and distribution activities of electrical energy, water and gas, and for the first time for base 2010, division 36 as well: Water collection, filtering and distribution, from section E in CNAE-2009. This indicator reflects the joint evolution of quantity and quality, eliminating the influence of prices.

The IPI is presented as the original series, and as the series adjusted for calendar effects. This adjustment is carried out in order to eliminate the influence on the number of working days and the number of holidays in the different Autonomous Communities, and thus be able to carry out homogeneous comparisons between the months of different years. The seasonality of the series is also corrected with the base 2010 (which now has been replaced by 2015).

The analytic work has been made with two datasets from National Statistics Institute (INE):

- 1975M1-2015M9: Monthly growth rates for original data[3].
- 1993M2-2015M9: Monthly growth rates for original data once the seasonal and calendar effects are eliminated)[4].

We will present some comparative results obtained by using both time series for IPI.

Although the economic crisis that began in 2008 caused significant setbacks in industrial production in the main economies of the euro area, in Spain the initial impact was of greater magnitude and, unlike other countries, there has not been an appreciable recovery from 2009. The contractive effect of the crisis has been more intense in our country, and recovery, also slower. The real state crisis has had a significant impact on industrial production. Building sector acts as a source of demand for industrial products that generates both direct and indirect carryover effects. In this sense, the intense contraction of residential construction in Spain caused a significant decline in industrial production, which became a loss of consumption and investment in equipment. The redirection gradual process of the industrial production towards foreign markets has been supported by an improvement of the competitiveness-price and cost of the sector. Foreign demand has partially offset the weakness of domestic demand in Spain; exports have contributed positively to manufacturing production. However, if Spanish exports of manufactures have been affected by expansive behavior in recent years, they have contributed, in a reduced way, to reducing the impact of the crisis on industrial production in Spain. The entry into recession of the Spanish economy at the beginning of 2012 showed new decreases in the IPI.

(III) Consumer Price Index (CPI) (Monthly Growth Rates) for the period 1961M1-2015M9 (INE)[5]: It is the rate of inflation in Spain based on the consumer price index or CPI for short. The Spanish CPI shows the change in prices of a standard package of consumer goods and services acquired by households in Spain. It is measured in terrms of the annual growth rate and in index, 2010 base year with a breakdown for food, energy and total excluding food and energy.

[3] http://www.ine.es/jaxiT3/Tabla.htm?t=26061
[4] http://www.ine.es/jaxiT3/Tabla.htm?t=26069
[5] http://ine.es/jaxiT3/Tabla.htm?t=10013

Over this period, prices in Spain experienced moderate inflation. However, certain factors, such as rising commodity prices in international markets, tax increases introduced as part of the fiscal consolidation package and an increase in certain regulated prices conditioned the inflationary trend.

The rise in oil prices that occurred since the end of 1973 by OPEC decision, which was repeated in an expanded form in 1979-1980 (although prices recorded the inverse evolution, very strong decline, in 1985), it influenced completely new domestic inflation. The sociopolitical transformations and the new conditions of labor relations ended up influencing the evolution of prices. However, the process of deceleration in the prices growth was very slow due to the persistence of formulas of social agreement to reconcile the wage moderation that was sought to achieve. A fall in the energy prices, throughout 1985, resulted in an important decrease in the growth rate of the CPI in 1985 and 1986. It is a last phase, which closes the 20th century and opens the 21st century, with a progressive fall of the inflation rate. The moderation in public expenditure and the reduction of the budget deficit, which had been maintained at a high level until 1995, supported the actions in search of price stability and on the path of reducing the price differential that the Spanish economy has been registering with respect to the economies of the EU. The energy crisis of the 2000s produced a strong inflationary trend. Spain reached in 2009 the lowest inflation rate in the last 40 years, there was for the first time deflation since there are recorded data. In October 2010, the economy continued to contract while inflation increased again. Between 2011 and 2012, prices rose by 3.5%. This increase, combined with austerity measures and high unemployment, negatively impacted the standard of living.

4. Wavelet Power Spectrum and density of OIL Price, IPI and CPI. Empirical findings and economic interpretation

The Wavelet Power Spectrum (WPS) characterizes the distribution of the energy (spectral density) of a time series across the two-dimensional time-scale plane, leading to a time-scale (or time-frequency) representation. Given the discrete wavelet transform W_m^x, then WPS is defined as the absolute-value squared of the wavelet coefficients, that is,

$$WPS = |W_m^x|^2 \tag{5}$$

WPS helps us understand the evolution of the variance of a time series at the different frequencies, with periods of large variance associated with periods of large power at the different scales. It can quantify the time evolution of the oscillatory modes and show when they are dominant.

It is clear that the different time-series have different characteristics in the time–frequency domain. Therefore, we illustrate the results from WPS for our economic variables and draw a suitable practical interpretation.

Each figure contains four parts: **(a)** represents the growth rate for the variable under consideration, displaying the transient dynamics for the Spanish economy; **(b)** the most important one, illustrates the WPS. On the horizontal axis, we have the time dimension. The vertical axis gives us the period cycles. The colour code for power ranges from blue (low power) to red (high power). The white lines show the maxima of the undulations of the wavelet power spectrum and provide an estimate of the cycle period. The thick black line shows the cone of influence, the contour designates the 5% significance level; **(c)** gives similar information because the Global Wavelet Power Spectrum (GWPS) is an average, over time, of the WPS; **(d)** gives the Fourier Power Spectral Density (PSD) that shows the strength of the variations (energy) as a function of frequency.

4.1. OIL Price

Figure 4 exhibits a stable behaviour in the oil price growth rate until the mid-1970s. A structural change due to the oil crisis of 1974 brought with it higher price volatility (variance). We conclude that after 1974 there is evidence of structural change in the oil price series due it became much more volatile. Looking at the WPS, between 1975 and 1980 the power at low and medium scales was high. In the late 1990s, low time-scale bands also show high power. At medium scales, the power of the price was high around 2010. This same information can be also seen in the GWPS. In fact, these results and comments are in line with [2] when considering only the data up to 2007.

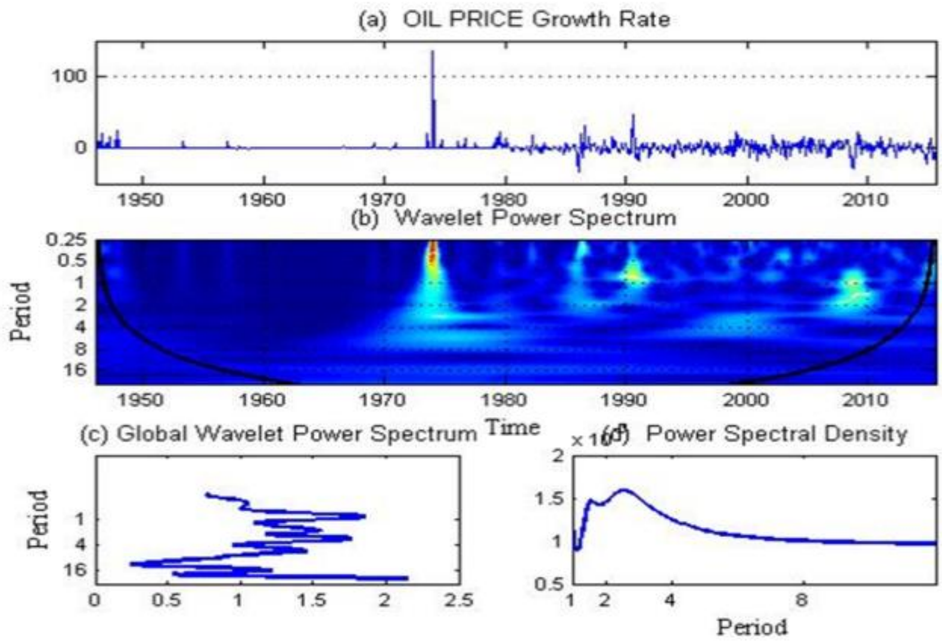

Figure 4. (a) OIL Price Monthly Growth Rate. (b) WPS. (c) DPWS. (d) Fourier PSD.

4.2. Industrial Production Index (IPI)

We consider two cases based on the available data:

- Case 1: The sample is shorter but the seasonal and calendar effects have been eliminated from the INE methodology.
- Case 2: We consider the original IPI data from INE, which result in a longer sample than in Case 1. Next, a deseasonalisation process by multiplicative effect is applied to obtain the time series data.

Figure 5 includes two options which differ in the way the monthly growth rates are calculated from the available INE data. The top (bottom) figure shows Case 1 (Case 2). Note that changes in the dynamics of the IPI variable in both cases are nearly impossible to identify in the two Figures (a). Looking at the WPS, the power at medium scales was high between 1990 and 2000 and more significate during 2005-2013, which is best seen in Case 2. In particular, a cycle of 2-4 year periodicity would be relevant to explain the total variance of the series for such time periods. In Case 2 is better appreciated that from 1985 the power spectrum at higher scales is more intensive; long term scale bands (8~12 years) from 1985 can explain the total variance of the series.

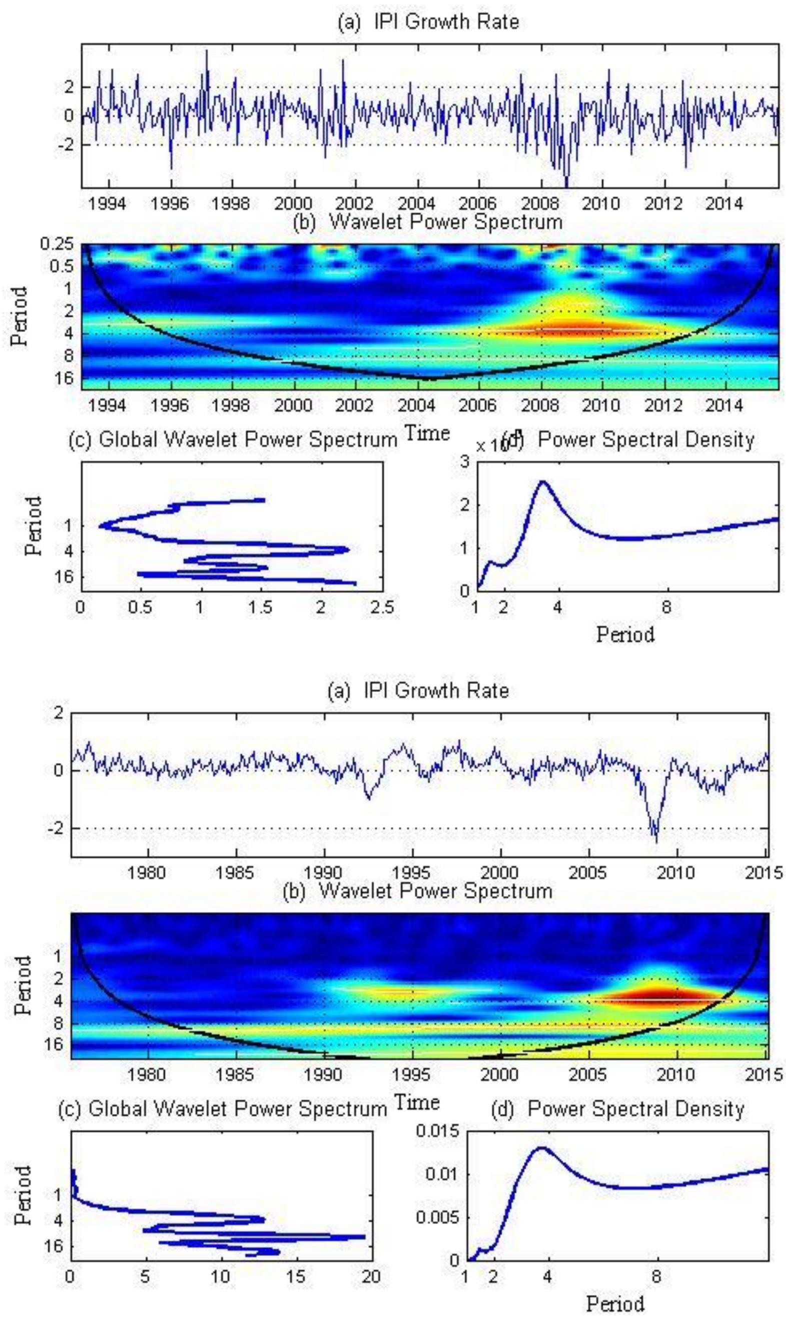

Figure 5. IPI data Case 1 (top figure) and Case 2 (bottom figure). (a) IPI Monthly Growth Rate. (b) WPS. (c) DPWS. (d) Fourier PSD.

4.3. Consumer Price Index (CPI)

In Figure 6 we are able to spot that before 1990 the inflation rate variance was relevant at short and medium scales (bands less than 4 years). After 2001, the variance of the inflation rate is again higher at very short scales, suggesting that we were facing very short term shocks to inflation. At high scales (8-16 year period cycle), the inflation rate variance was relevant suggesting that we were facing long term shocks to inflation.

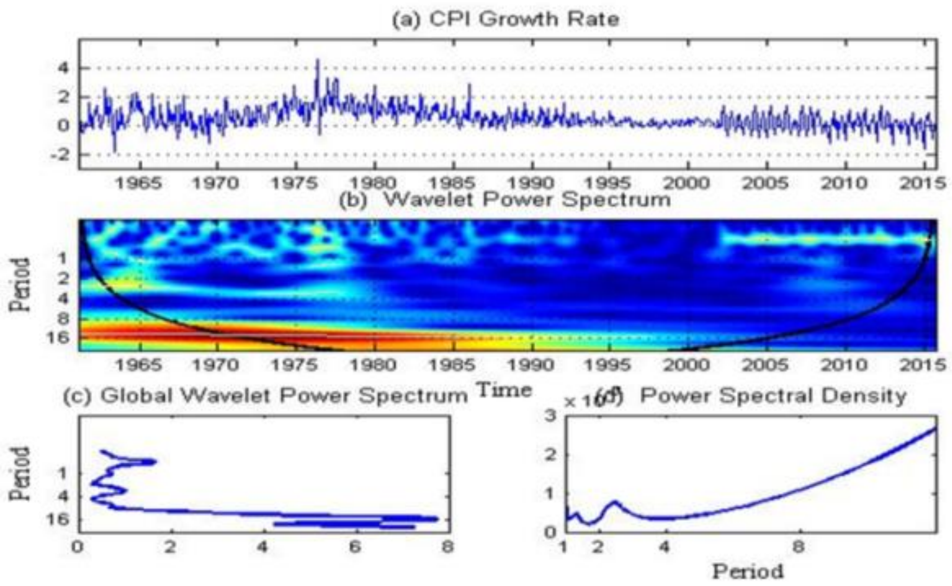

Figure 6. (a) CPI Monthly Growth Rate. (b) WPS. (c) DPWS. (d) Fourier PSD.

5. Cross Wavelet Coherency and Phase-Difference: Co-movements and possible causal relationships between OIL Price, IPI and CPI. Empirical findings and economic interpretation

The Cross Wavelet Coherence and Phase-Difference are efficient tools for studying the co-movements, i.e. to capture the dependence structure across different timescales and the causality relationships between signals. These tools are also based on the DWT.

Given two signals, x and y, with the respective DWTs being W_m^x and W_m^y, the Cross Wavelet Transform is defined by

$$CWT(x, y) = W_m^{xy} = W_m^x\,(W_m^y)^* \tag{6}$$

where $()^*$ represents the complex conjugate.

Then, the Cross Wavelet Power is

$$CWP(x, y) = \left|W_m^{xy}\right| \tag{7}$$

The normalization between x and y, known as Cross Wavelet Coherency, is given by

$$CWC(x, y) = \frac{|W_m^{xy}|}{|W_m^x||W_m^y|} \tag{8}$$

And their position in the pseudo cycle is called Phase. The Phase Difference given by

$$\phi_{xy} = Arctan\left(\frac{Im(W_m^{xy})}{Re(W_m^{xy})}\right), \tag{9}$$

provides information on the relative position between x and y, that is, the delay, or synchronization, between their oscillations .

The following theoretical example (González-Concepción et al., 2017) is illustrative for better understanding the process that occurs over time and frequency. Let us consider the variables x and y given by

$$x_t = \begin{cases} \cos\left(2\pi\frac{t}{12} + \alpha\right) + \varepsilon_t & t \le 36 \\ cos\left(2\pi\frac{t}{12} - \alpha\right) + \varepsilon_t & 36 < t \le 72 \\ cos\left(2\pi\frac{t}{12} + 2\alpha\right) + \varepsilon_t & 72 < t \le 108 \\ cos\left(2\pi\frac{t}{12} - 2\alpha\right) + \varepsilon_t & 108 < t \le 144 \end{cases} \qquad y_t = cos\left(2\pi\frac{t}{12}\right) + \varepsilon_t \tag{10}$$

with $\frac{\pi}{4} < \alpha < \frac{\pi}{2}$ and where ε_t is i.i.d. $N(0,1)$. If we simulate these variables, the corresponding time series are shown in Figure 7. It is not easy to identify the common cycles and the delays or how their cycles relate to each other.

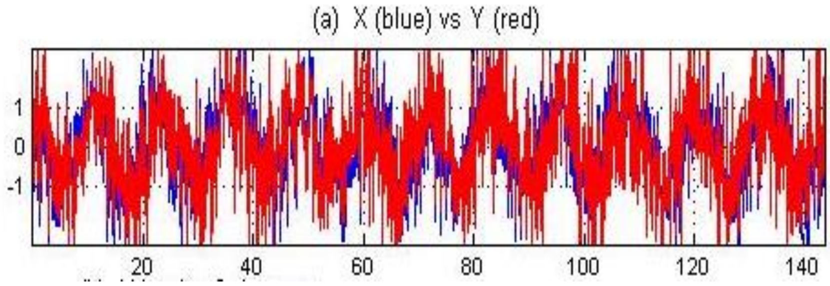

Figure 7. Original time series in the theoretical example.

However, from the theoretical expression of the determinist part (Figure 8), we can observe the series move IN Phase (they move in the same direction or positively related) in the first half (initially *x leading* and then *y leading*) . They are OUT of Phase (they move in the opposite direction or negatively related) in the second half (initially *y leading* and then *x leading*).

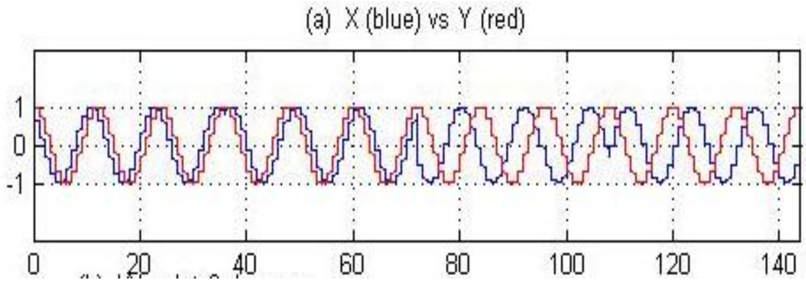

Figure 8. Determinist part in original time series.

The wavelet tools we have raised can be helpful to reveal these co-movements and causal relationship between *x* and *y*.

Figure 9 shows the CWC, where we have different regions based on a coherency level (from blue zone to red zone, corresponding to low to high coherency). The coherency is high in every period in time, except in certain periods and at times 36, 72 and 108.

In 11~12 year frequency bands, we can also see the *x*-Phase (blue line), *y*-Phase (green line) and Phase-Difference (red line). We employ the phase difference tool to investigate the dependence and causality relationships. We can make out four zones in the red line that can be interpreted as shown in the diagram in Figure 10:

- Zone A: Red line in [-π/2,0], between times 0-36, means that *x* and *y* are IN Phase with *x leading.*
- Zone B: Red line in [0,π/2], between times 36-72, means that *x* and *y* remain IN Phase, with *y leading*.

- Zone C: Red line in [-π,-π/2], between times 72-108, means that *x* and *y* are OUT of Phase, with *y leading*.
- Zone D: Red line in [π/2,π], between times 108-144, means that *x* and *y* remain OUT of Phase, with *x leading*.

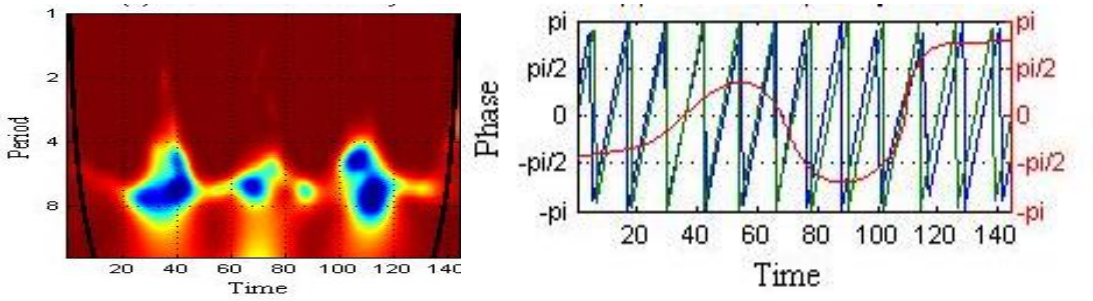

Figure 9. CWC, Phases and Phase- Difference in theoretical example.

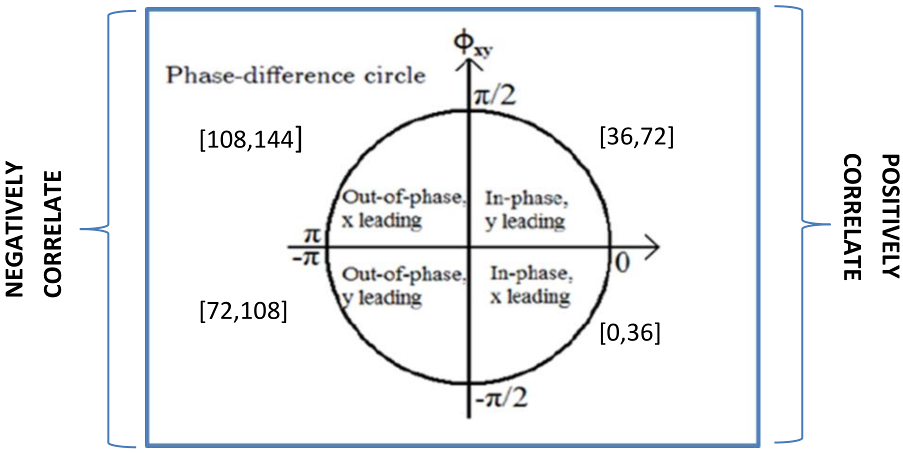

Figure 10. Diagram from [7] applied to our theoretical example.

To investigate the interdependence among economic series, we show some figures obtained by computing the CWC for each pair, the Phases and the Phase-Difference using Morlet wavelets. We consider the *period* as the reference variable.

These figures contain four parts: **(a)** shows *x* and *y* (Monthly Rates), *x* (blue line) and *y* (red line); **(b)** Presents Wavelet Cross Coherency, with low Coherency (blue zone, lower dependence between the series) and high Coherency (red zone, a significant relation); **(c)** and **(d)** give the Phases in frequency band indicated, *x*-Phase (blue line), *y*-Phase (green line) and the Phase-Difference between *x* and *x* (red line). The black contour designates the 5% significance level.

5.1. OIL Price versus IPI

In Figure 11 we have on the top the growth rates for both series. On the left, it appears the coherency between the pair of variables. On the right, we present two graphs: the phase-difference between the two time-series calculated for the 4-8 years frequency-band and the analysis in the longer run, i.e., for 8-12 years frequency-band.

As concerns the wavelet coherency, Figure 11 shows that coherency regions vary widely in small periods (intermittent high power regions). The analysis reveals strong wavelet coherence (strong interdependence) from 4-year cycles and especially from 8-year cycles (medium and long term), indicating a high correlation at these scales. The significant area covers the entire sample period at this scale. However, between 4–and 8-year cycles, regions with low coherency are observed except around 1990. The Phase-Difference gives us more details about the delay between oscillations of two time-series.

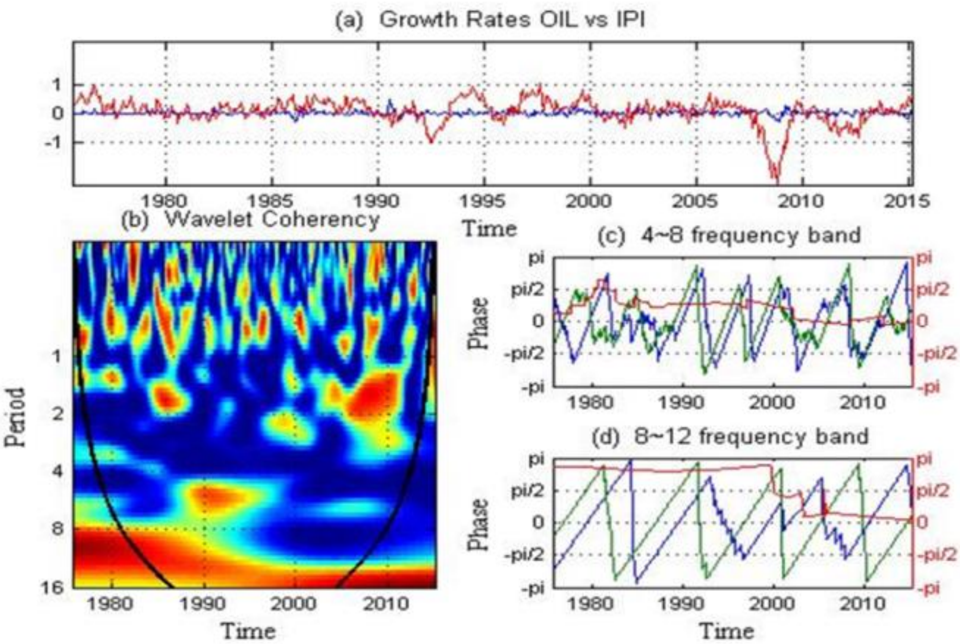

Figure 11. x=OIL Price, y=IPI (Case 2)

Looking at the 4~8 year frequency band, the Phase-Difference (ϕ_{xy}) moves in $[0,\pi/2]$ except in 1980, where it is in $[\pi/2,\pi]$. Therefore, most of the period, OIL and IPI move IN Phase that is, there is a phase relationship between both series, with IPI *leading*, indicating that oil price changes were demand-driven. It happens except in 1980, when they are OUT of Phase (there is an anti-phase relationship, analogous to negative covariance). It means that a negative relationship is found between both series, with OIL *leading*, i.e. oil-price increases led to increases in the industrial production, capturing the negative effects of oil price shocks.

Considering the 8~12 frequency band, the Phase-Difference is in $[\pi/2,\pi]$ before 2000 and in $[0,\pi/2]$ from then on. It indicates that both variables are OUT of Phase with OIL as the *leading* variable until 2000. From that date, both are IN Phase with IPI *leading*, indicating that oil price changes that have occurred after 2000 were demand-driven.

5.2. CPI versus IPI

Figure 12 shows the wavelet coherence plot. In particular, a highly variable coherency can be observed in low periods (small frequency bands). There is high coherency around 2-4 year periods between 2005 and 2010. The phase-difference gives us more information. In both frequency bands, the variables are IN Phase with IPI l*eading*, except in 4~8 frequency band between 1980 and 1985, where CPI is *leading.*

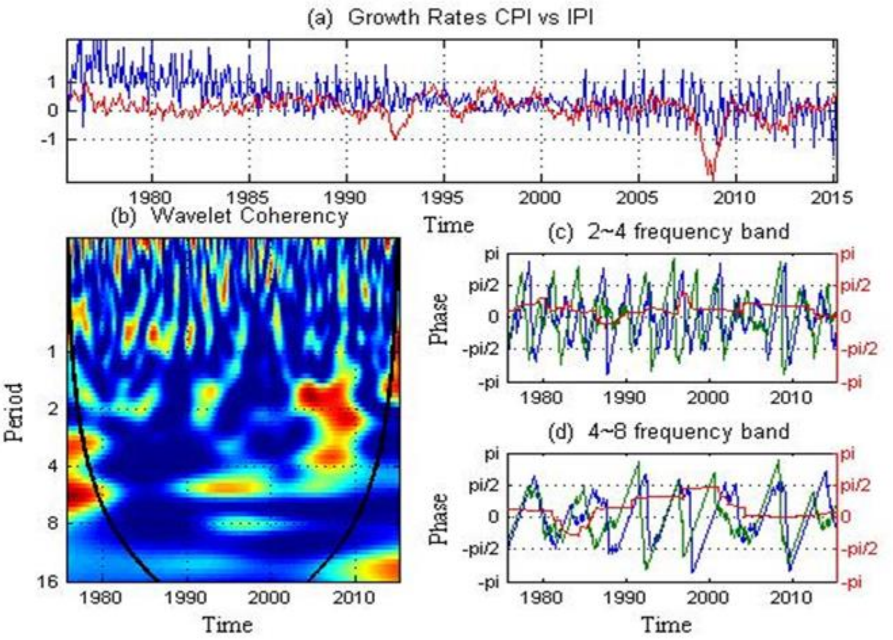

Figure 12. *x*=CPI, *y*=IPI (Case 2)

5.3. OIL Price versus CPI

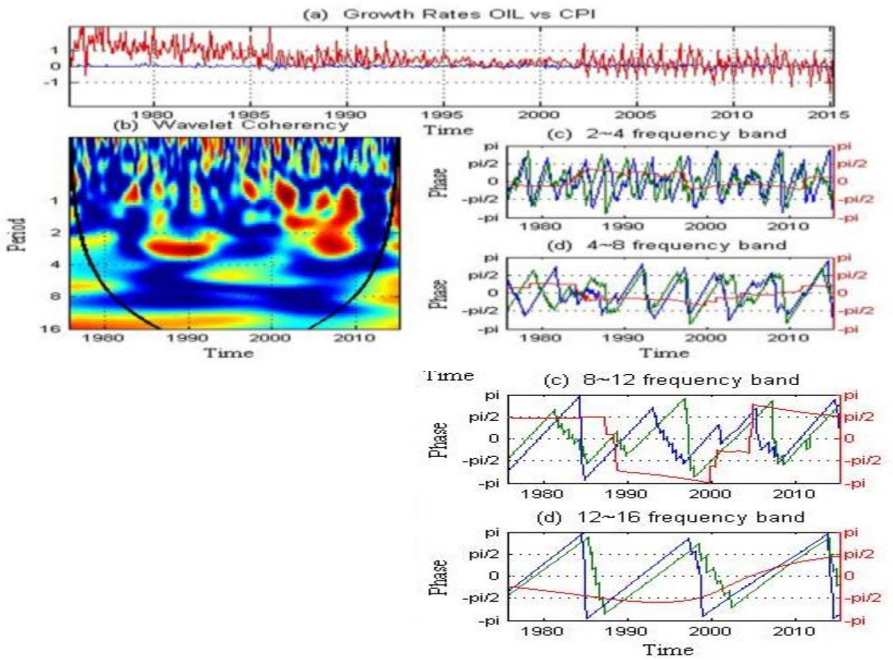

Figure 13. *x*=OIL Price, *y*=CPI

In Figure 13 we estimate the coherency between the oil price series and inflation. We found highly variable coherency levels in most of the period, with somewhat more stability in low coherency in medium-high bands. High coherency can be also detected in periods from 2 to 4 years around 1990 and in periods from 1 to 4 years between 2005 and 2010.

This is also verified by observing the phase of the oscillations, as well as their phase-difference. In summary, in 2~4 year bands both series move IN Phase, with OIL *leading* before 1985 and after 2000. Between those two years, CPI is *leading*. The opposite happens in 4~8 year bands. At medium scales, we have evidence that in 8~12 year time-scale bands the series move OUT of Phase, except from 2000-2005. In this case, CPI is the *leading* variable between 1990 and 2000 and OIL is the *leading* one before 1990 and after 2005. Between 2000 and 2005, a speedy transition occurs from OIL *leading* to CPI *leading*, with both series IN Phase. If we focus in high 12~16 year frequency bands, both series move IN Phase except between 1990 and 2000, indicating that OIL is the *leading* variable before 1990 and CPI is *leading* from 2005. Between 1990 and 2000, both series are OUT of Phase with CPI *leading.*

6. Conclusions

We use the wavelet tools to investigate economic cycle's synchronization among relevant macroeconomic variables in Spain. The wavelet power spectrum (WPS) illustrates the evolution of the variance of a time-series at the different frequencies; the wavelet coherency (CWC) illustrates the correlation coefficient in the time–frequency space; the information on the delay between the oscillations of two time-series, i.e., lead–lag relationships is provided by the phase-difference.

When considering the interrelationship between the three relevant Spanish macroeconomic variables chosen, some of our conclusions are similar to those expressed in [5], where six economic variables are studied without considering the IPI.

Once again, our findings show significant changes around 2008 in the relationship between these variables. This could indicate a delicate border between the periods before and after the global financial crisis. The shift in the dependency between economic variables when crossing this border was also analysed for certain variables in [9].

For instance, the influence of the Oil Price on different economies has been of interest to numerous researchers [2-5, 10, 11…]. In particular, in the German macroeconomic, the relationship between OIL Price, IPI and CPI was considered by [11] using CWC, with the author finding ambiguity between OIL Price and IPI. In the German economy, the Phase-Difference between OIL Price and CPI is IN Phase and OUT of Phase over time, with inflation being the leading variable in most of the cases.

The direct contribution of changes in OIL Price to the CPI in Spain and the euro zone has been analysed in [10]. Our findings, based on the analysed data, support the lead-lag relationship of the OIL Price. This is due to Spain is an oil importer and, in contrast to other advanced economies, has always been more sensitive to changes in the crude oil price.

Some interesting results can be derived to compare our findings and those included in [2]. In particular, the time series for the OIL price, until 2007M12, is common in both studies. They also analyse the relationship between OIL Price and IPI and CPI for the US economy. Specifically, comparative results of the US economy with our findings for the Spanish economy reveal interesting common patterns. Structural breaks can be observed for both economies when comparing OIL Price and IPI. At the 4~8 year frequency band, a structural break occurred in the mid-1980s, when the series were IN Phase, with IPI *leading*: OIL price changes that occurred after mid-1980s were more demand driven. At the 8~12 year frequency band, after the end of 1995, another change occurred that substantiates some recent empirical results (oil price changes that have occurred after the mid-1990s were demand driven). These results are broadly in line with those of several authors who concluded that, approximately since the Asian crisis, demand-driven oil price shocks have become more important than supply-side induced shocks. Putting all this information together, one could conclude that demand driven oil price shocks became important around 1985, and became even more important after some period around 1995. As concerns OIL Price versus CPI, the relationship between oil price increases and inflation was very stable in both studies. Oil price increases lead inflation increases across time and across frequencies. In the US economy, a tight monetary policy in the 1980s proved to be successful, with a decrease in the inflationary impact of oil price shocks. During the 1990s, the inflationary impacts of oil price increases were also very well contained. In [2] the CPI to IPI relationship is not considered, and we could not find results for the US economy to conduct a comparative study using wavelet analysis.

Continuing with the US economy, [4] found, using wavelet analysis, that stock returns lead aggregate economic activity on long-term scales. And in the short term, that the relationship between stock returns and output is not fixed over time. Moreover, that stock market returns lead output on the longest scales, which suggests that it is primarily investors with longer time horizons who are linked to macroeconomic fundamentals in their investment activity.

A comparison with the results obtained by [3] for the German economy shows certain similarities in terms of the relationship between OIL Price and CPI, meaning that in most cases inflation is the leading variable. Also, if we consider the relationship between OIL Price and IPI in the Spanish economy, we find an alternation between IN Phase and OUT of Phase.

We have focused on a better knowledge of the economic variables through the wavelet techniques in a period which covers the financial crisis that broke out in 2008. As noted [12], volatility or turbulence (type, speed, volume and scale) has intensified since the crisis began. And uncertainty increases over the conditioning events of any prediction. The economic and geopolitical interrelationships have become more complex and the potential consequences more ambiguous. In these more volatile, uncertain, complex and ambiguous environments, it no longer matters only what is likely to happen, but also what is possible, even if its probability is low.

An interesting task for future work would be to deal with forecasting models that integrate wavelets techniques to provide forecasts over different horizons. We could refer to wavelet-based prediction procedures. They obtain low and high frequency components after decomposition of the original time series to be used as input variables to forecast economic variables.

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