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THE ORNSTEIN–UHLENBECK PROCESS TO MODEL THE DEPOSIT VOLUME OF NON-MATURING ASSETS IN COLOMBIA

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RESUMEN: La comprensión precisa de los factores de riesgo de las diferentes instituciones de depósito es la clave para su operación sostenible. En este documento, analizamos dos enfoques estocásticos para modelar activos sin fechas de maduración (NMA) empleando un proceso Ornstein-Uhlenbeck que puede usarse para la evaluación del riesgo de liquidez e interés de las cuentas de depósitos a la vista en los bancos. Detallamos las especificaciones, los parámetros y los resultados de la simulación de los modelos. Además, examinamos los patrones regulares, a lo largo del año, del comportamiento del volumen de depósitos en cuentas de depósitos a la vista en Colombia, en concordancia con los resultados de otros investigadores en diferentes países. Finalmente, encontramos que se debe incorporar un término de tendencia en el modelo para capturar el crecimiento de la serie.

*Palabras clave***:** Depósitos a la vista, Gestión del riesgo de liquidez, proceso Ornstein–Uhlenbeck.

ABSTRACT: The accurate comprehension of the risk drivers of different depository institutions is the key to their sustainable operation. In this paper, we analyze two stochastic approaches to model Non-Maturing Assets (NMAs) employing an Ornstein–Uhlenbeck process that can be used for the evaluation of the liquidity and interest risk of savings accounts in banks. We detail the models' specifications, parameters, and simulation results. Furthermore, we examine the regular patterns, throughout the year, of the behavior of the volume of deposits into saving accounts in Colombia, in line with the results of other researchers in different countries. Finally, we found that a trend term should be incorporated into the model to capture the growth of the series.

*Keywords***:** Demand deposits, Liquidity risk management, Ornstein–Uhlenbeck process.

1. Introduction

Banks borrow and lend money from and to multiple agents with different purposes and investment intentions, using a variety of products and implicit interest rates. Such institutions seek to maintain a positive spread of the assets they invest in and the cost of their funds (Kalkbrener & Willing, 2004). This is because their funds can be withdrawn by customers at any time as their products have no specific contractual maturity, as described by Entrop et al. (2009), which implies that banks regularly invest in assets with different levels of liquidity by transforming liquid deposits into long-term assets.

During the 2007–2009 financial crisis, banks seriously compromised their liquidity. Throughout the early liquidity phase, many banks, while maintaining adequate levels of capital, ran into difficulties by not managing their liquidity prudently (Basel, 2011). According to (Cornett et al., 2011), such crisis offered a unique challenge to financial institutions and regulators to improve their liquidity risk management; furthermore, it brought along significant fiscal and monetary responses by the US government that produced the risk of inflation (Nawalkha & Soto, 2009).

As described by Cornett et al. (2011), liquidity is the cornerstone of financial intermediation: banks need liquidity to supply credit demands and withdrawals from depositors. Liquidity depends on information, as explained by Goodhart (2008): under the assumption of market completeness where the information of a complete set of contingent securities is available to all agents participating in this market, liquidity problems are not generated, since the assets can be traded at their fundamental value. Vento and La Ganga (2009) define liquidity risk as the risk that a financial intermediary does not have sufficient liquid resources to meet its obligations or must obtain such liquidity at excessive costs

The Basel Committee on Banking Supervision (BCBS, 2013) promotes the short-term resilience of the liquidity risk profile of banks to survive a significant stress scenario. The Committee aim to ensure that the bank has adequate cash, or equivalent assets, to meet its liquidity needs for a 30-calendar day. This implies the use of an adequate tool to measure such liquidity needs and give some clues on how to define a policy regarding a minimum liquidity level. In this framework, the main goal of risk analysts is to estimate cash flows of future periods and identify the maximum possible variations to appropriately capture indicators such as Cash Flow at Risk (Волошин and Микита, 2020). Measuring liquidity involves the estimation of future cash flows for all assets and liabilities, such as interest payments, deposit into saving accounts, and bonds, among other assets (Feilitzen, 2011).

In this paper, we study a way to represent the dynamics of saving accounts, which can be considered Non-Maturing Assets (NMAs), adopting a stochastic approach. The principal issue of this kind of assets is the embedded option that clients may exercise at any moment and the unhedged volume risk (Frauendorfer & Schürle, 2007). Fortunately, the data available for this kind of analysis is enough to present statically significance. The stochastic modeling approach for NMAs has been studied by Jarrow and van Deventer (1998), Kalkbrener and Willing (2004), Dewachter et al. (2006), Frauendorfer and Schürle (2007), Nyström (2008), Cipu and Udriste (2009), Paraschiv and Schürle (2010), Musakwa (2013), Blöchlinger (2015), Džmuráňová and Teplý (2015), Henningsson and Skoglund (2016), Grundke and Kühn (2020), Волошин and Микита (2020); most of them, based on Vasicek (1977).

In this work, we analyze two stochastic models of NMAs[1] that can be used for the evaluation of liquidity and interest risk of savings accounts in Colombian banks. We also study the convenience of employing an Ornstein–Uhlenbeck process to describe the dynamics of NMAs in Colombia in terms of volume of deposits and the logarithm of such volume[2]. Modeled as a mean-reverting process, the deposit volume always returns to some equilibrium level: the stronger the mean reversion, the faster the deposits will go from an extreme value to their equilibrium (the expected mean). Moreover, its long-term variance converges to a constant and does not increase indefinitely; thus, deposit volume maintains its movements within some range nearby its long-term mean.

The paper is structured as follows. Section 2 presents a description of the characteristics and dynamics of deposit volume. Section 3 introduces the mathematical model that represents the stochastic behavior of

[2] The logarithmic transformation allows the smoothing of the series' volatility.

deposit volume, while Section 4 examines data of Colombian saving accounts. Section 5 focuses on process estimation and simulation. Finally, Section 6 offers some concluding remarks about the results we found and recommendations for future studies.

2. Deposit volume

The volume each client deposits into a bank account is seen as an amount of money that has been lent to the bank. Such accounts are deemed Non-Maturing Assets, as depositors are free to, at any time, deposit or withdraw capital from their deposit account. Therefore, they do not have a predetermined maturity date, a source of uncertainty for future cash flows (Henningsson & Skoglund, 2016). Rational decision-making by customers is reflected in the behavior of non-maturity deposits and depends on 2 factors: the value received and the value perceived. The volume of an NMA position moves as clients react to interest rates; for example, when interest rates are low, clients choose fixed-rate loans.

Deposit volume modeling must consider some characteristics, such as those described by Frauendorfer and Schürle (2007) for NMAs. The driver of deposit volume is customers' economic behavior, that is, when an investment option appears, the demand for deposits into saving accounts tends to decrease. When interest rates are low, clients try to anticipate their investment decisions, and when such rates are high, they postpone said decisions. Unfortunately, the embedded prepayment and withdrawal options of NMAs represent a volume risk.

Jarrow and van Deventer (1997) explain demand as a process with no arbitrage opportunities for individuals. They demonstrate that the Net Present Value of demand deposits into a bank can be calculated by Equation (1), where D_t represents the volume of demand deposits; i(t), the net servicing cost; B(t), the value of the money market account; and E(·), the expected conditional value at t=0.

$$V_D(0) = D_0 + E\left(\sum_{t=0}^{\tau-2} \frac{D_{t+1} - D_t}{B(t-1)}\right) - E\left(\frac{D_{\tau-1}}{B(\tau)}\right) - E\left(\sum_{t=0}^{\tau-1} \frac{i(t) \cdot D_t}{B(t-1)}\right) \tag{1}$$

This expression assumes, and that is an important finding or our research, that the aggregate demand deposits depend only on the evolution of the term structure of default-free rates. Therefore, the relationships between deposit volumes and interest rates should be investigated to establish whether rate changes impact customer behavior. In turn, Jarrow and van Deventer (1997) represent deposit volume as a linear function of market interest rate, r_t: $D_t = \beta_0 + \beta_1 \cdot r_t$, where β_0 and β_1 are constants. A similar conclusion was reached by Paraschiv and Schürle (2010) when they found that the series of deposit rates and market rates are cointegrated.

Furthermore, the long- and short-term randomness of deposit volumes must, hence, inherit the dynamics of short-term interest rates. Considering Vasicek's model as an appropriate way to describe the stochastic structure of interest rates, a mean-reverting stochastic process must be established to represent deposit volume. This is coherent with the modeling proposed by Kalkbrener and Willing (2004) and the findings of Paraschiv and Schürle (2010): the speed of the adjustment back to equilibrium grows with the increase in the magnitude of client rate's disequilibrium.

3. Mathematical model

3.1. *Volume*

The first model we use in this study is proposed by Kalkbrener and Willing (2004). They assume a normal distribution of volume increments and model the volume process $V(t)$, for all $t \in [0, \infty)$, as the sum of a deterministic linear function $f_V(t)$ and an Ornstein–Uhlenbeck process whit zero mean X_t, as shown in (2).

We assumed that, for the initial time $t = 0$, the deposit volume and the Ornstein–Uhlenbeck process had a known value: $V(0) = V_0$ and $X_{t=0} = x_{0,}$, respectively.

$$V(t) = f_V(t) + X_t$$

$$dX_t = -\kappa_V X_t \cdot dt + \sigma_V \cdot dW_t \tag{2}$$

where $f_V(t) = \beta_o^V + \beta_1^V \cdot t$ is a linear trend that represents the mean growth of Volume. Parameters β_o^V and β_1^V are determined by a linear regression, as in the work by Kalkbrener and Willing (2004). In equation (2), $\kappa_V > 0$ is the speed of adjustment, dW_t represents a variation to standard Brownian motion, and $\sigma_V > 0$ is the short-term volatility. The explicit solution to the deposit volume (3) can be obtained using Itô's Lemma.

$$V(t) = f_V(t) + \left(V_0 - f_V(0)\right) \cdot e^{-\kappa t} + \sigma_V \int_0^t e^{\kappa(s-t)} dw(s) \tag{3}$$

with the expected value and variance that can be obtained in accordance with Appendix A.

$$E[V(t)] = f_V(t) + \left(V_0 - f_V(0)\right) \cdot e^{-\kappa t}$$

$$Var[V(t)] = \frac{\sigma_V}{2\kappa_V}\left(1 - e^{-2\kappa t}\right) \tag{4}$$

That is, in the long run, the expected value and variance are deterministic, i.e., they do not depend on the initial conditions of Volume V_0.

$$\lim_{t\to\infty} E[V_t] = f_V(t)$$

$$Var[V_t] = \frac{\sigma_V^2}{2\kappa_V} \tag{5}$$

3.2. *Log-Volume*

A second kind of model is created from the natural logarithm of the deposit volume, $v(t) = \log(V(t))$. We assume, for that purpose, that it can be written as (4), where

$$v(t) = f_v(t) + Y_t \tag{6}$$

$$dY_t = -\kappa_v Y_t \cdot dt + \sigma_v \cdot dW_t \tag{7}$$

where $\kappa_v > 0$, $Y(0) = v(0) - f_v(0) = Y_0$, and $f_v(t) = \beta_o^v + \beta_1^v \cdot t$ is a linear trend that represents the mean growth of the Log-Volume. The following section describes in depth the deterministic function we implement in order to capture the seasonality and structural changes in Volume and Log-Volume.

Hence, the volume has a lognormal distribution with mean

$$E[V_t] = \exp\left(E[v(t)] + \frac{1}{2} Var[v(t)]\right)$$

$$E[V_t] = \exp\left(f(t) + Y_0 e^{-\kappa t} + \frac{\sigma_v^2}{4\kappa}(1 - e^{-2\kappa t})\right) \tag{8}$$

and variance

$$Var[V_t] = \exp(2E[v(t)] + Var[v(t)])\left[\exp(Var(v(t)) - 1\right]$$

$$Var[V_t] = \exp 2\left(f(t) + Y_0 e^{-\kappa_v t} + \frac{\sigma_v^2}{4\kappa}(1 - e^{-2\kappa_v t})\right) \cdot \left[exp\left(\frac{\sigma_v^2}{2\kappa}(1 - e^{-2\kappa_v t})\right) - 1\right]. \tag{9}$$

In order to estimate the parameters for the Stochastic Differential Equation (SDE) for X_t y Y_t using discrete observations, we need to express them fully in a discrete form, according to Agudelo et al (2018):

$$X_j = (1 - \kappa) \cdot X_{j-1} + \epsilon_j \tag{10}$$

For j from 2 to the number of observations. Where residuals $\epsilon_j$are i.i.d. normally distributed.

3.3. *Deterministic function f(t)*

To implement models (1) and (6), we specify a deterministic function $f(t)$, which captures both the seasonal pattern and the structural changes in savings accounts' behavior after 2014 (see Appendix B) in Colombia. Such function represents all the relevant and predictable dynamics of deposit volume, and its selection depends on the specific characteristics of the data we have in order to meet all the assumptions of the linear regression model. We modified the deterministic function used by Kalkbrener and Willing (2004), adding dummy variables for each month D_i, from January (D_1) to November (D_{11}), and a dummy variable D_S for the structural change in t_S, which modifies long-term growth since January 2015. The deterministic functions of Volume and Log-Volume have the same specifications, but their coefficients vary. The function $f(t)$ in (11) is a generalization for $f_V(t)$ and $f_v(t)$.

$$f(t) = \beta_0 + \beta_1 \cdot (1 - D_s) \cdot t + \beta_2 \cdot D_s \cdot t + \sum_{i=1}^{11} D_i \cdot M_i$$

$$D_s = \begin{cases} 1;\ t > t_s \\ 0; t \le t_s \end{cases} \tag{11}$$

Such specification for the deterministic function is applied to detrend the time series in order to compute different parameters of the Ornstein–Uhlenbeck process for Volume and Log-Volume. Parameters β_0, β_1, β_2, and M_i are estimated from a historical time series of deposit volumes. Subsequently, we use a segmented linear regression, as suggested by Gujarati (2009), which is a type of linear regression where the slope of the model varies from β_1 to β_2 since the date of the structural change (see Figure 1).

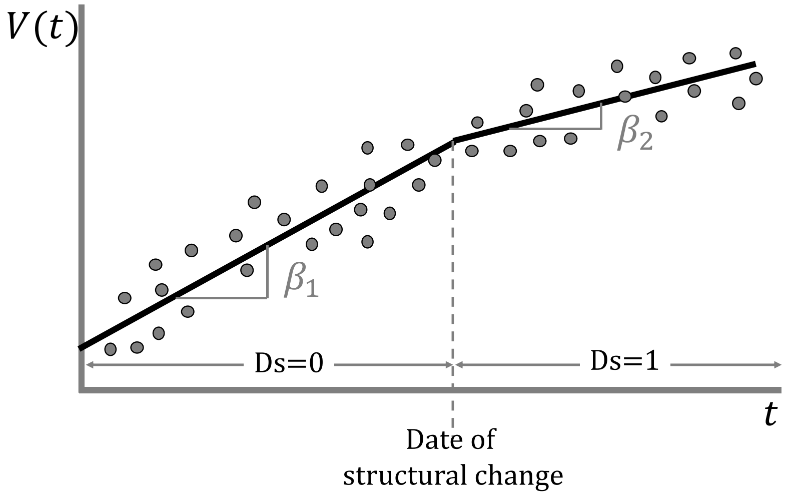


Figure 1. Segmented linear regression.

4. Data description

The database consists of 228 monthly observations of the aggregate deposit volumes of the current accounts of the banks report to the regulator in Colombia.[3] From 2001 to 2014, the financial information was prepared under the local accounting regulations; from 2015 onwards, the financial statements have been prepared following the Accounting and Financial Reporting Standards issued by the International Accounting Standards Board (IASB).

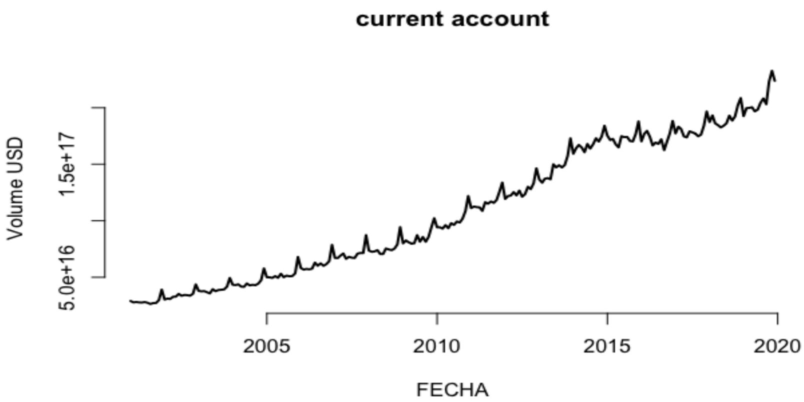


Figure 2. Observed Deposit volume

Figure 2 shows the monthly evolution of the aggregate volumes of deposits in current accounts for the analysis period. Seasonal components are observed, and the trend is accelerating until December 2014 and from January 2015 presents a period of growth deceleration until the end of 2016, where the accelerated series is again observed. Within the analyzed sample, 60% of the institutions correspond to international banks which concentrate more than 80% of the banking assets.

3 We used information available from the Colombian bank regulator in *Información financiera con fines de supervisión* in *Estados financieros (COLGAAP)* and *Información financiera con fines de supervisión* (IFRS). The variable *Volúmenes de saldo de cuenta corriente* was taken from *Superintendencia financiera de Colombia* at https://www.superfinanciera.gov.co/inicio/informes-y-cifras/cifras/establecimientos-de-credito/informacion-por-sector/bancos-60775

Table 1 presents the descriptive statistics of Volume and Logarithm of Volume. Panel A lists the results of the whole dataset; Panel B, of the data before the date of the structural change; and Panel C, of the data after such change. the descriptive statistics show that after the structural change the average of the volumes of current account deposits increases.

Table 1. Descriptive statistics of monthly deposit volumes in Colombia.

Series	Number of observations	Percentile 5	Percentile 95	Mean	Median	Standard Deviation	Skewness	Kurtosis
Panel A. All series (from Jan. 2001 to Dec. 2019)								
V(t)	228	8,239,739,058	53,453,740,209	29,476,338,165	26,518,547,376	15,887,313,299	0.20	1.61
log (Vt)	228	22.83	24.70	23.93	24.00110547	0.628872993	-0.37	1.83
Panel B. Before structural change (until Dec. 2014 for Volume and until Aug. 2014 for Log-Volume)								
V(t)	168	7,685,797,833	44,481,060,126	22,175,297,81	19,464,008,272	11,548,434,352	1	2.3
log (Vt)	168	22.76	24.52	23.68	23.69	0.54	-0.09	1.9
Panel C. After structural change (since Dec. 2014 for Volume and since Aug. 2014 for Log-Volume)								
V(t)	60	45,264,434,615	56,548,364,887	49,919,250,679	48,744,711,261	4,104,805,655	1	3.9
log (Vt)	60	24.54	24.76	24.63	24.61	0.08	0.89	3.4

5. Results

Table 2 summarizes the model's estimation of the deterministic functions. Note that all the parameters we use are statically significant in a 99% confidence interval. Additionally, in accordance with the adjusted R^2, the deterministic function of Volume explains 97.77% of the variations in the observed data and 98.96% in the Log-Volume version.

For the estimation of seasonal effects, the month 12 (December) is taken as the reference level. May, September, and October are the months in which the volumes of current accounts are lower, while December, January, and February are the months in which the volumes of deposits are higher.

Table 2. Estimated F(t) for Volume and Log-Volume.

Coefficients	F(t) for Volume				F(t) for Log-Volume			
	Value	StdError	tvalue	Pr(>|t|)	Value	StdError	tvalue	Pr(>|t|)
β_0	5.99E+09	6.77E+08	8.858	0.000%	22.8700	0.0180	1272.468	<0.0001
β_1	2.29E+08	3.95E+06	58.112	0.000%	0.0110	0.0001	104.998	<0.0002
β_2	2.37E+08	2.50E+06	94.595	0.000%	0.0094	0.0001	141.431	<0.0003
D1	-3.02E+09	8.07E+08	-3.742	0.410%	-0.1009	0.0214	-4.709	0.0019%
D2	-2.88E+09	8.07E+08	-3.567	0.392%	-0.1056	0.0214	-4.931	0.0005%
D3	-3.08E+09	8.06E+08	-3.824	0.294%	-0.1140	0.0214	-5.32	0.0002%
D4	-3.49E+09	8.06E+08	-4.325	0.007%	-0.1262	0.0214	-5.89	0.0000%
D5	-4.29E+09	8.06E+08	-5.327	0.000%	-0.1564	0.0214	-7.303	0.0000%
D6	-3.32E+09	8.06E+08	-4.114	0.210%	-0.1180	0.0214	-5.509	0.0002%
D7	-3.87E+09	8.06E+08	-4.807	0.001%	-0.1475	0.0214	-6.889	0.0000%
D8	-3.62E+09	8.06E+08	-4.487	0.029%	-0.1428	0.0214	-6.671	0.0000%
D9	-4.14E+09	8.06E+08	-5.133	0.001%	-0.1595	0.0214	-7.449	0.0000%
D10	-3.74E+09	8.06E+08	-4.637	0.009%	-0.1525	0.0214	-7.125	0.0000%
D11	-2.54E+09	8.06E+08	-3.149	0.097%	-0.1126	0.0214	-5.259	0.0000%
Adjusted R^2	0.9777				0.9896			

The estimated parameters of the Ornstein Uhlenbeck process are given in Table 3. The mean reversion parameter κ for V(t) is 0.08 and for logV(t) is 0.2, this implies that for V(t) a deviation from the long-term mean is corrected by 8% for the following period and for logV(t) the correction will be 20% (which implies that the reversion speed to the mean of logV(t) is greater than speed reversion of V(t)). The coefficient of determination of the mean reversion model for V(t) is 0.864 and for logV(t) is 0.6143. These determination coefficients explain the variability of the residues remaining from the estimation of the parameters of the deterministic function, which implies an additional explanation of the model, on the variability V(t) and logV(t).

After the structural change in 2014, the growth rate of the deposit volume and Log-Volume decreased in Volume. Before the structural change, the time slope was 0.0109 monthly and, after it, 0.0094. The Ornstein–Uhlenbeck process estimation is presented in Table 3.

Table 3. Ornstein–Uhlenbeck process estimation for X(t) and Y(t).

Parameter	X(t) for Volume				Y(t) for Log-Volume			
	Estimate	StdError	tvalue	Pr(>|t|)	Estimate	StdError	tvalue	Pr(>|t|)
$1-\kappa$	0.92	0.02428	37.9	<2e-16	0.80	0.04217	18.97	<2e-16
κ	0.08				0.2			
σ	8.82e+08				0.03987			
Adjusted R^2	0.864				0.6143			
AIC	9998.55				-815.62			

After we found the parameters, we ran a stochastic simulation of the process we established. Figure 3 presents the simulation plots of a thousand trajectories, the 95% confidence interval, the 99% confidence interval, and the date of the structural change. The series V(t) and logV(t) are within the confidence intervals and the simulated probability distributions are close to the normal distribution.

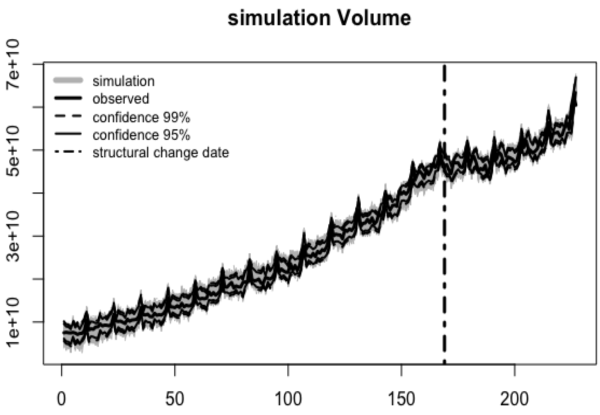
6. Conclusions

In this paper, we studied the convenience of using an Ornstein–Uhlenbeck process to describe the dynamics of Non-Maturing Assets in Colombian savings accounts in terms of deposit volume and the logarithm of deposit volume. The proposed model has deterministic and stochastic components. Additionally, it was necessary to detrend the series to capture the long-term growth, a structural change in 2014, and seasonality, which describe the behavior of this market and provide some reference points that regulators and banks must consider in order to monitor and control liquidity risk.

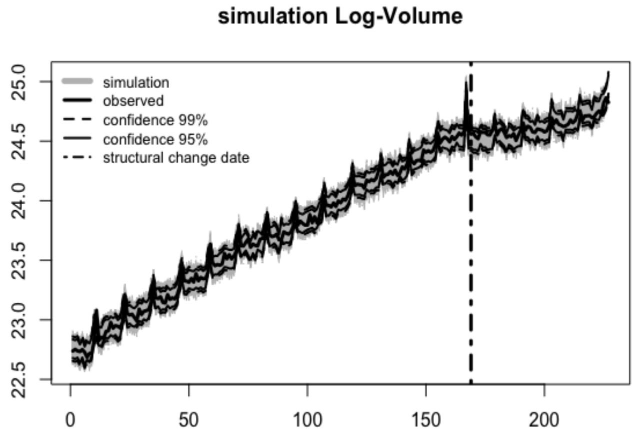
As deposit volume is modeled by a mean-reverting process, it always returns to some equilibrium level: the stronger the mean reversion, the faster deposits will go from an extreme value to their equilibrium (the expected mean). Long-term variance converges to a constant and does not increase indefinitely; thus, deposit volume maintains its movements within some range near its long-term mean. Therefore, the dynamics of deposit volume are considerably different from those of the stock market, where the expected variance increases over time to infinity. In accordance with this finding, risk managers should select an appropriate model to describe the expected mean of a number of future periods under analysis. This is because the levels of risk exposure depend not only on the variance but on the expected value as well.

We found that the assumptions of an Ornstein–Uhlenbeck process are satisfied in the Log-Volume model to a greater extent than in its Volume counterpart; the residuals after the process of volume estimation are not independent. Due to this lognormal characteristic of the volume of deposits into savings accounts, CVaR should be used instead of VaR to measure liquidity risk.

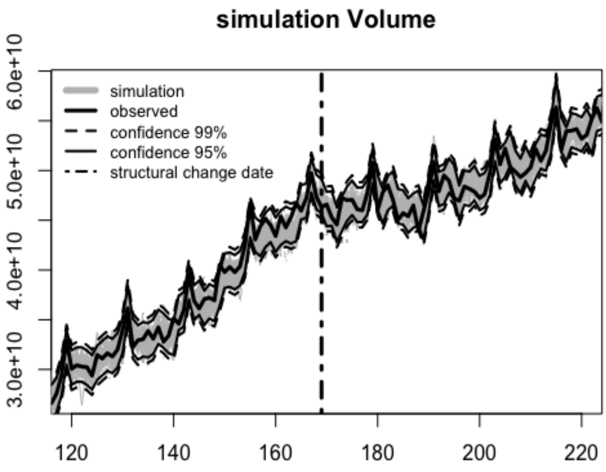
Further research could investigate the determinants of the structural changes since 2014 in savings account deposits by Colombian market participants; one of their possible causes is the emergence of alternative investments products. Moreover, a multivariate model could be formulated to capture the relationships among interest rates, deposit volume, GDP, inflation, and other factors to implement more accurate risk measurements coherent with short-term economic behavior. In addition, since our analysis employed aggregate-level data, individual banks should be considered to understand the specific customer response to both systematic and idiosyncratic factors.



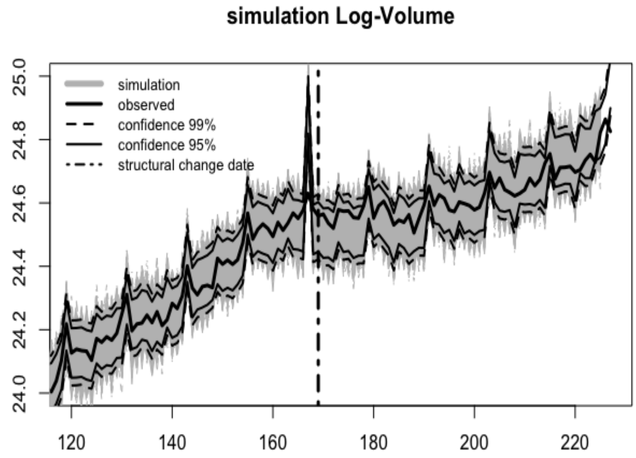
(a) Volume



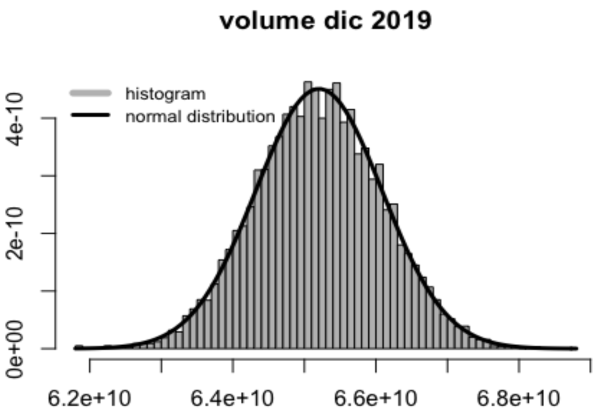
(d) Log-Volume



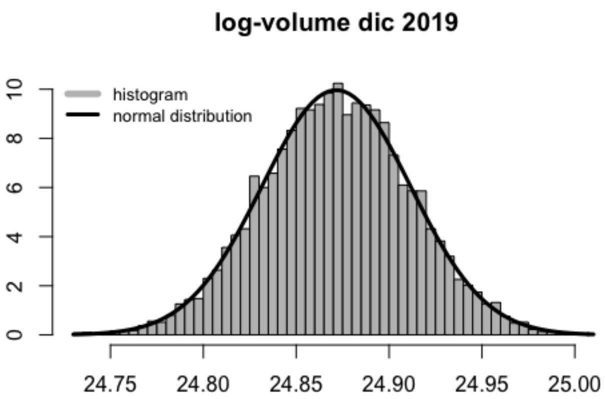
(b) Volume since 2011



(e) Log-Volume since 2011



(c) PDF Volume from Volume model, last period



(f) PDF Volume from Log-Volume model, last period of time

Figure 3. Deposit volume simulations.

The behavior of the analyzed series after the structural change indicated by the descriptive statistics implies a decrease in the risk inherent to the variations of the volumes in terms of liquidity (mainly those that imply retirement flows). the variability of the volumes of deposits in current accounts is mainly explained by their trend, seasonality, and mean reversion.

References

1. Kalkbrener. Michael & Willing. Jan. (2004). Risk management of non-maturing liabilities. Journal of Banking & Finance. Vol 28. Pages: 1547–1568. 10.1016/S0378-4266(03)00131-6.
2. Entrop O., Wilkens, M., & Zeisler, A. (2009). Quantifying the interest rate risk of banks: assumptions do matter. European financial management, 15(5), 1001-1018.
3. Cornett, M. M., McNutt, J. J., Strahan, P. E., & Tehranian, H. (2011). Liquidity risk management and credit supply in the financial crisis. Journal of Financial Economics, 101(2), 297-312.
4. Nawalkha, S. K., & Soto, G. M. (2009). Managing interest rate risk: The next challenge? Journal of Investment Management, 7(3), 86-100.
5. Goodhart, C. (2008). Liquidity risk management. Banque de France Financial Stability Review, 11, 39-44.
6. Vento, Gianfranco & La Ganga, Pasquale. (2009). Bank Liquidity Risk Management and Supervision: Which Lessons from Recent Market Turmoil? Journal of Money.
7. BCBS (2013). Basel III: The Liquidity Coverage Ratio and liquidity risk monitoring tools. [online] bis.com. Available at: http://www.bis.org/publ/bcbs238.pdf [Accessed 7 Mar. 2018].
8. Frauendorfer, K., & Schürle, M. (2007). Dynamic modelling and optimization of non-maturing accounts. Liquidity Risk Measurement and Management: A Practitioner's Guide to Global Best Practices, 327-359.
9. Feilitzen, H. (2011). Modeling non-maturing liabilities.
10. Jarrow, R. A., & Van Deventer, D. R. (1998). The arbitrage-free valuation and hedging of demand deposits and credit card loans. Journal of Banking & Finance, 22(3), 249-272.
11. Dewachter, H., Lyrio, M., & Maes, S. (2006). A multi-factor model for the valuation and risk managment of demand deposits. National Bank of Belgium working paper, (83).
12. Nyström, K. (2008). On deposit volumes and the valuation of non-maturing liabilities. Journal of Economic Dynamics and Control, 32(3), 709-756.
13. Cipu, E. C., & Udriste, S. (2009). Estimating non-maturity deposits. Relation, 2, 1.
14. Paraschiv, F., & Schürle, M. (2010). Modeling client rate and volumes of non-maturing accounts.
15. Musakwa, F. T. (2013, April). Measuring bank funding liquidity risk. In Actuarial Approach for Financial Risks Colloquium in Lyon.
16. Blöchlinger, A. (2015). Identifying, valuing and hedging of embedded options in non-maturity deposits. Journal of Banking & Finance, 50, 34-51.
17. Džmuráňová, H., & Teplý, P. (2015). Duration of Demand Deposits in Theory. Procedia Economics and Finance, 25, 278-284.
18. Henningsson, P., & Skoglund, C. (2016). A framework for modeling the liquidity and interest rate risk of demand deposits.
19. Vasicek, O. (1977). An equilibrium characterization of the term structure. Journal of financial economics, 5(2), 177-188.

20. Agudelo, Gabriel A. & Franco, Luis C. & Franco, Luis E. Mar (2018). Procesos Estocásticos Aplicados a las Finanzas. Optimal Research Group. Medellín. ISBN:978-958-59824-6-8.
21. Gujarati. D. N. (2009). Basic econometrics. Tata McGraw-Hill Education.
22. Grundke, P., & Kühn, A. (2020). The impact of the Basel III liquidity ratios on banks: Evidence from a simulation study. The Quarterly Review of Economics and Finance, 75, 167-190.
23. Bank for International Settlements. (2011). Basel III: International Regulatory Framework for Banks.
24. Волошин, Ігор Владиславович, and Ігорович Волошин Микита. 2020. "ESTIMATING the Worst Scenario of Net Cash Outflow of Non-Maturity Deposits Based on Quantile Regression." The Banking University Bulletin, no. 1 (37): 124–30.

Appendix A

First, we have the closed form (3). Here, we show the derivative of expected value and the variance of deposit volume, taken the conditional expected value for $V(t)$ in t=0

$$E[V(t)] = E\left[f(t) + \left(V_0 - f(0)\right) \cdot e^{-\kappa t} + \sigma \int_0^t e^{\kappa(s-t)} dw(s)\right]$$

$$\Rightarrow E[V(t)] = E[f(t)] + E\left[\left(V_0 - f(0)\right) \cdot e^{-\kappa t}\right] + \sigma_V E\left[\int_0^t e^{\kappa(s-t)} dw(s)\right]$$

$$\Rightarrow E[V(t)] = E[f(t)] + E\left[\left(V_0 - f(0)\right) \cdot e^{-\kappa t}\right]$$

$$\Rightarrow E[V(t)] = f(t) + \left(V_0 - f(0)\right)$$

For the conditional variance:

$$Var[V(t)] = E\left[V(t) - E[V(t)]\right]^2$$

$$\Rightarrow Var[V(t)] = E\left[\sigma \int_0^t e^{\kappa(s-t)} dw(s)\right]^2$$

$$\Rightarrow Var[V(t)] = \sigma^2 \int_0^t e^{2\kappa(s-t)} dw(s)$$

$$\Rightarrow Var[V(t)] = \frac{\sigma}{2\kappa}\left(e^{2\kappa(t-t)} - e^{2\kappa(0-t)}\right)$$

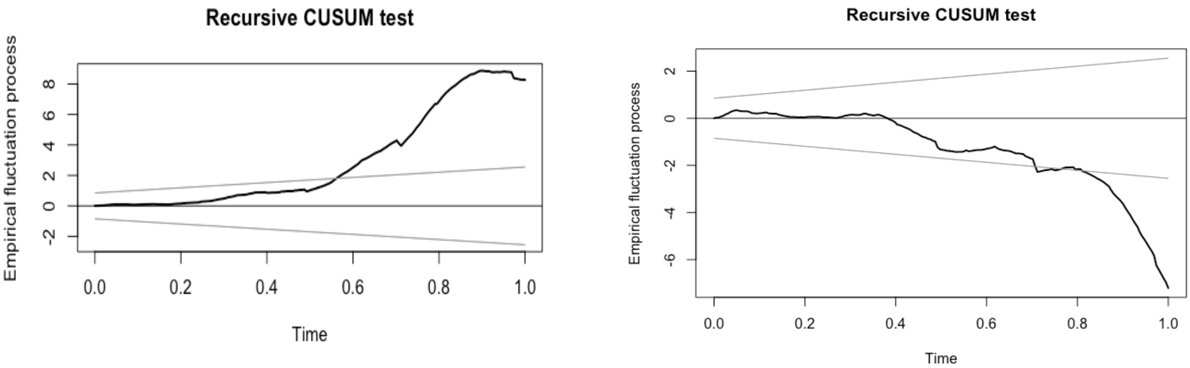
$$\Rightarrow Var[V(t)] = \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa t})$$

Appendix B

The recursive CUSUM test considers a cumulative sum of standardized residuals $\tilde{e}_i$:

$$W_n(t) = \frac{1}{\tilde{\sigma}\sqrt{\eta}} \sum_{i=k+1}^{k+t\eta} \tilde{e}_i$$

Under the null hypothesis, the limiting process of the empirical fluctuation process $W_n(t)$ is Wiener Process. Under the alternative hypothesis, there is only a single structural change in all the series. Figure 4 shows the recursive CUSUM test the Volume and Log-Volume series.



(a) Test for Volume (b) Test for Log-Volume

Figure 4. Recursive CUSUM test.

Table 4 summarizes the CUSUM test. We found a structural change because the null hypothesis was rejected since December 2014 for Volume and since August 2014 for Log-Volume.

Table 4. CUSUM test of Log-Volume.

Deterministic function	Date of structural change	F-statistic	Pvalue	Ho
$f_V(t)$	December 2014	2.403	1.85e-10	Rejected
$f_v(t)$	August 2014	3.170	2.2e-16	Rejected